Ghana Journal of Education: Issues and Practice (*GJE*)

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Ghana Journal of Education: Issues and Practices (GJE)
Vol. 1, December 2015, pp. 1-28

Why are non-routine mathematics word problems difficult?: Lessons from pre-service basic school teachers in Ghana

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Abstract

This study sought to contribute to the literature on why non-routine word problems in Mathematics often seem difficult for learners. Three hundred and sixty-nine Primary and Junior High School teacher trainees from three Colleges of Education in Southern Ghana participated in the study. A non-routine mathematics word problem achievement test was administered to the teacher trainees, after which 18 (out of the 369) were interviewed to explain their processes. The difficulties encountered by participants were analysed using Newman's (1977/1983) Error Analysis as the theoretical framework. The results revealed that the pre-service teachers generally had weak proficiency in non-routine word problem solving. The majority of participants could not solve problems at the Junior and Senior High School levels. Implications of the findings for pre-service teacher preparation at the College of Education level in Ghana and countries that have similar mode of teacher education are provided.

Key words: Non-routine mathematics problems, Word problems in mathematics, Difficulties with non-routine mathematics problems, Pre-service teachers.

Introduction

Problem solving has become an emerging theme for the past three decades, gaining prominence in research and in mathematics education (Kilpatrick, 1985; Törner, Schoenfeld & Reiss, 2007). The call for its recognition and inclusion in mathematics teaching and learning continues to intensify worldwide (National Council of Teachers of Mathematics, 2000; Posamentier & Krulik, 2008; Stacey, 2005). Literature suggests that proficiency in problem solving promotes conceptual understanding of mathematical concepts. Hiebert, Carpenter, Fennema, Fuson, Diana, Murray, Olivier and Human (1997), for example, argue that "if we want students to understand mathematics, it is more helpful to think of understanding as something that results from solving problems, rather than something we teach directly" (p. 27). Teachers' proficiency in problem solving also affects students' opportunity to learn mathematics and their attitudes to mathematics. Lappan and Briars (1995) contend that "there is no decision that teachers make that has a greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the task with which the teacher engages the students in studying mathematics" (p. 138). Other researchers have also argued that by engaging students in solving everyday real life problems and those that are non-routine in nature, they learn to become flexible thinkers and good problem solvers in practical situations (see for example, Polya, 1973). A growing body of studies have therefore engaged with issues relating to routine and non-routine mathematical problem solving (Reed, 1999; Yeo, 2009).

This paper contributes to the understanding of why solving of non-routine word problems in mathematics is difficult for students by looking at Ghanaian pre-service teacher trainees' proficiency in non-routine problem solving in order to ascertain how proficient they are before they start teaching as newly qualified teachers. In Ghana, the mathematics syllabus for Primary, Junior High School and Senior High School levels require that students are taught to apply their knowledge, develop analytical thinking skills, generate ideas and creative solutions, and address everyday mathematical situations. Problem solving and investigations are therefore at the core of the Ghanaian mathematics curriculum (MOE, 2010, 2012a, 2012b). Teachers are expected to include appropriate and realistic problems and mathematical investigations that will require the use of mathematical processes and also provide opportunities for students to explore mathematical ideas (MOE, 2010, 2012a, 2012b). This requirement in the pre-tertiary school mathematics curriculum in general, and the Primary and Junior High School mathematics curricula in particular, is important for the total development of students who go through mathematical training in Ghana. However, despite several interventions implemented to improve students' performance in mathematics over the years (for example, the STME programmes by the Government of Ghana and other donor agencies and the designation of the then ten Colleges of Education but now nineteen as Mathematics and Science Colleges), Ghanaian students continue to perform poorly in mathematics, especially on non-routine word problems at the basic school level (that is, Primary and Junior High School levels), (Amoah, 2016; Intsiful, 2014; Mullis, Foy & Arora, 2012).

Since teachers play a critical role in students' acquisition of mathematical skills and the development of strong mathematical knowledge base, they cannot be isolated from students' achievement (Osiakwan, Wilmot & Sokpe 2014; UNESCO, 2010; Wilmot, 2009). This is because teachers are expected to help students acquire problem solving skills and strategies that are needed for development of creative thinking and intellectual curiosity in mathematics. We argue that to be able to train the right calibre of teachers who can engender the needed intellectual curiosity among their students at the basic school level in Ghana, proficiency in problem solving is paramount. One cannot agree more with the drafters of the current pre-service teacher education mathematics curriculum when they stated that the mathematics courses for pre-service teacher education at the Colleges of Education (CoEs) in Ghana are expected to give exposure and to also equip prospective teachers with problem solving skills and knowledge of problem solving strategies (IoE, 2005, 2014). However, no study has been carried out to ascertain whether the curriculum is indeed achieving this aim. This study therefore contributes to literature on how the mathematics curriculum at the Colleges of Education in Ghana prepares prospective Primary and Junior High School teachers to handle non-routine problems in mathematics.

Studies looking at pre-service teacher trainees' proficiency in problem solving in mathematics is not new. Elsewhere in the United States of America, Rosli, Capraro, Goldsby, Gonzalez, Onwuegbuzie and Capraro (2015), for example, investigated middle-school pre-service teachers' problem solving and problem posing proficiency in mathematics and found that "even though the majority of the pre-service teachers understood the problem statement, most of them were not able to write an equation or describe in words the generalised solution for any number of steps" (p.340). Unlike Rosli et al. (2015), the present study did not consider problem posing by the participating pre-service teachers. Instead, it focused on getting participants to engage in problem solving sessions using non-routine word problems in algebra at the Primary, Junior High and Senior High school levels and investigating their proficiency in problem solving using the Newman's approach. The study focused on algebra because it forms the foundation for the development of the other domains of mathematics such as geometry and calculus, to mention just a few. In other words, our perspective is that weak mastery of algebra could affect students' performance in mathematics in general; hence the need to focus on algebra.

Specifically, the current study was designed to investigate first the achievement of pre-service teacher trainees in algebraic non-routine mathematical problems, and second, the difficulties experienced by pre-service teacher trainees in solving the algebraic non-routine mathematical problems. Although this study was done in Ghana, the findings should be valuable source of literature to overseas researchers and experts who are interested in educational development in developing countries, as well as those interested in similar studies in such countries.

Theoretical Framework

This study employed Newman's (1977, 1983) classification of sources of problem- solving difficulty as basis for exploring pre-service teacher trainees' challenges and reasons behind their solution paths. Newman's (1983) model was used as a framework for this study because it provides a comprehensive stage-wise procedure to analyze sources of difficulty of mathematics problem solvers in solving mathematical problem-solving tasks. Hence it provided the researchers the opportunity to ascertain the various sources of the teacher trainees' difficulty in handling the non-routine problem-solving tasks.

The feature of the model is that, it condenses the process of solving mathematics word problems into five levels which are "reading", "comprehension", "transformation", "processing" and "encoding" as shown in Figure 1. In her model, Newman (1983) suggested that errors occur in the interaction between the question and the person who is attempting to solve the problem. Newman argues that when a learner produces an incorrect answer to a question, the error or misrepresentation resulting in that answer may have occurred at one of five stages in the process of solving that problem. The student may have misread the question (reading error), or may have misinterpreted it (comprehension error). Even if the student understands the problem, s/he may incorrectly transform it into mathematical language. Despite a correct transformation, an incorrect method may have been used to solve the problem (process error). In circumstances where all the above mentioned errors are evaded, the solver can still make errors by wrongly encoding the answer (encoding error) or may have difficulty with explaining or verifying the answer (verification error).

This study adapted the five-level interview format because the aim and focus of the study was to understand pre-service teacher trainees' error patterns. The components of the five-level format were (a) reading the problem (reading stage); (b) interpreting it (comprehension stage), (c) transforming the problem into mathematical equation (transformation stage), (d) selecting a strategy to solve it (processing stage). The processing stage also involved solving the problem using the selected strategy and (e) verification of the answer or looking back in order to ensure that the solution addresses the problem it sets out to address. This involved answering verification questions from the interviewer based on their responses presented in their worksheet.

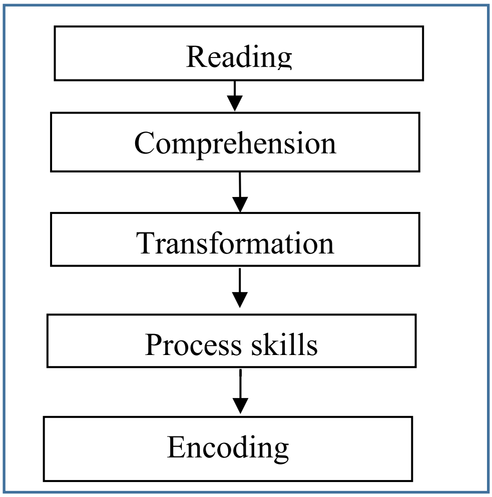

Figure 1. Interview format (Newman, 1977, 1983)

The Design of the Study

According to Brews (2001), the rationale of every research is to contribute to the body of knowledge that shapes and guides academic as well as practice disciplines. The choice of methodology depends on the behaviour to be studied and the intended focus of the study (Mertens, 2003). The present study employed a mixed-methods design. The mixed-methods design was preferred because this paradigm systematically combines both quantitative and qualitative methods to aid understanding of the purpose and focus of the study.

In this study, an aspect of the mixed-methods paradigm, the Sequential Explanatory Design (see Creswell, 2012), which is characterised by the collection and analysis of quantitative data followed by the collection and analysis of qualitative data was employed since the study sought to ascertain the achievement of the teacher trainees in non-routine problem solving and also explore the difficulties they faced solving such problems.

In the quantitative phase, a survey of the achievement of the research participants was carried out. In this phase, therefore, an achievement test instrument was used to identify and classify pre-service teacher trainees' performance and strategies. The findings from the quantitative analysis helped to determine the research participants for the qualitative phase. The qualitative data in this study were used for explanatory as well as for exploratory purposes. It was used to explain the quantitative findings and to explore it more deeply. In the qualitative phase, interviews were used to explore trainees' difficulties in solving the non-routine task and the reason(s) for their choice of strategies. The interview allowed the pre-service teacher trainees to articulate their thoughts and to verbalize their actions which guaranteed an insight into their thinking processes. Listening to pre-service teacher trainees' verbal explanations exposed their thinking patterns for interpretation and allowed for identification not only of the reasons behind their particular actions but also their misconceptions. The process promoted more self-reflection. Detailed explanation of the data collection procedure is presented under the research procedure section.

Instruments used in the study

Two instruments were developed and used to collect data for the study. They were a mathematical non-routine achievement test and an interview guide. The achievement test consisted of four sections. The first section elicited the biographical data of the respondents. The second tested trainees' performance in algebraic non-routine problem at the Primary level, while the third and fourth sections tested their performance in algebraic non-routine problems at the Junior High School and Senior High School levels. The items were adopted from already validated items such as mathematics textbooks, Trends in International Mathematics and Science Study (TIMSS) achievement tests, past questions of Basic Education Certificate Examinations and West African Senior Secondary Certificate Examinations, organised by the West African Examinations Council (WAEC). In order to preserve content validity, the test items were prepared in consultation with four senior mathematics tutors from two Colleges of Education located outside the research locale. The national mathematics teaching syllabi for Primary, Junior High School and Senior High School were also consulted in preparing the achievement test. To ensure and preserve construct validity, the instruments were pilot-tested in a College of Education in the Greater Accra Region. For the internal consistency estimates of the test instrument, the split-half and the reliability coefficients respectively were calculated taking into consideration two underlying assumptions; (a) the halves must have almost equal standard deviations and (b) the halves must be alike in content. It was found that the two halves had similar standard deviations (Std. Dev. for first half = 8.4 and Std. Dev. for second half = 8.7), an indication that the two halves were identical with respect to content. For the split-half coefficient, the Equal-length Spearmen-Brown was determined as both halves included equal number of items. The Equal-length Spearmen-Brown coefficient calculated was 0.76 and the reliability coefficient for the whole test was 0.8, implying the items were very reliable.

The second instrument was an interview guide, which was developed using Newman's Error Analysis (NEA) Guidelines (Newman, 1983, 1977) as a framework. The interview guide was semi-structured. This allowed for the use of prompts such as "explain more", "go ahead" and "how" or "why" where necessary to get more insights into the difficulties trainees faced in solving the problems. The interview guide was developed by the researchers and validated by pilot-testing it in a College in the Greater Accra Region to ensure that they elicited valid response.

Research Procedure

Prior to the field work, permission was sought from the principals and heads of mathematics department of the participating Colleges of Education in the Central Region. In each of the Colleges, the purpose of the study was explained to the trainees, their consents were sought before the selection of the research participants and the administration of the achievement test. On the whole, the data collection process took three weeks.

A multi-stage sampling procedure was used in selecting the research participants. The first stage involved purposive selection of all second year pre-service teacher trainees pursuing Three-Year Diploma in Basic Education programme from the three Colleges of Education in the Central Region of Ghana. The structure of the three-year teacher training programme in Ghana is such that the teacher trainees spend two years on campus to study content and pedagogy related courses and the third on doing practicum in schools of attachment. The second years were selected because they had just finished taking all their content and pedagogy related courses. They were therefore considered to be the most suitable for this study.

The second stage involved the selection of a sample of 369 teacher trainees (out of a total of 837) from the three Colleges of

Education for the quantitative phase of the study. These participants were selected using the stratified random sampling procedure. At this stage, participants were presented with six problems to solve individually in 60 minutes. To ensure anonymity, participants were given index numbers and requested to write these, instead of their names on the paper. In addition, they wrote the name of their college, programme of study, gender and age. After the test, the worksheets of participants were scored.

At the third stage (that is, following the administration and scoring of the test items), 18 out of the 369 participants comprising a mix of those who performed poorly in the test (that is, obtained very low scores) and those who performed well in the test (that is, obtained average scores or higher), from all the three colleges were selected using the purposive sampling procedure. Six were selected from each of the three Colleges. These were interviewed to elicit information on how they solved the problems and difficulties they encountered solving the problem. Each of the interview sessions lasted between 45 and 60 minutes.

Data Analysis Procedure

The quantitative data generated from the achievement tests were analysed using frequency counts, and descriptive statistics involving means and standard deviation. The analysis was carried out using the SPSS software. The data collected from interviews and worksheet of the trainees were analysed qualitatively. The interviews were transcribed, and explored to ascertain a general sense of it, and then coded for description. Extracts from participants' worksheets were also scrutinized and analysed qualitatively to identify their strategies which were then summarized under common themes. The themes or patterns that came up were presented as narrative descriptions with some illustrative examples. For the purpose of analysis, participants from College A were coded SA1, SA2, SA3, …, SA114. Participants from College B were coded SB1, SB2, SB3, …, SB130 while those from College C were coded SC1, SC2, SC3, …, SC125.

Results and Discussion

The results of the study are presented based on the two research questions that guided the study.

What is the achievement of pre-service teacher trainees in algebraic non-routine mathematical problem solving?

Table 1 presents the overall performance of pre-service teacher trainees on the algebraic non-routine problem-solving test. It also highlights the performance by College of Education. Results from Table 1 show that, pre-service teacher trainees generally performed poorly on the test. The overall average performance on the test was 13.07 (out of 36) with a standard deviation of 7.30. The overall average performance was far lower than the pass mark of 18 out of 36 (50%). A look at the results by College show that the overall average performance by each of the Colleges was also far below the pass mark of 18 out of 36 (50%). The minimum score of trainees in each of the Colleges was zero and maximum was 32 out of 36 (89%) for Colleges B and C, and 31 out 36 (86%) for College A. This gives a range of 32 (89%) and 31(86%) for Colleges B and C, and College A respectively, an indication of very high levels of variability in the scores of trainees. This high levels of variability is also exemplified by the high standard deviations associated with each of the mean scores.

Table 1
Overall Performance of Pre-service Teacher Trainees

Variable	Number	Minimum (out of 36)	Maximum (out of 36)	Mean (out of 36)	Std. Dev
College A	114	0.0	31.0	13.3	7.3
College B	130	0.0	32.0	13.1	7.4
College C	125	0.0	32.0	12.8	7.3
Overall	369	0.0	32.0	13.1	7.3

Table 2 presents a summary of pass rate of pre-service teachers based on the category of the problem-solving test items, that is, Primary, Junior High School (JHS) and Senior High School (SHS). The pass mark was set at 50% of the overall/maximum score for each of the categories of the problem. The pass rate was obtained by finding the percentage of students who obtained six out of twelve or better (i.e. 50% and above) in each category. Results in Table 2 show that the majority (58.9%) of the trainees measured up to the Primary school level, an indication that more than two-fifths (41.1%) did not measure up to the Primary school level in non-routine problem solving. More than a half

(52.6%) did not measure up to the JHS level in non-routine problem solving, with the overwhelming majority (98.7%) not measuring up to the SHS level. The mean score of trainees on the SHS items was less than 1 out of 12 (i.e. 6.7%). The high standard deviations associated with the mean scores, especially for SHS level items confirms the observation that there were very high levels of variability in scores among the trainees.

Table 2
Performance of Pre-service Teacher Trainees Based on Category of Problem

Problem Category	Number	Pass Rate (%)	Minimum (out of 12)	Maximum (out of 12)	Mean (out of 12)	Std. Dev
Primary	369	58.9	0.0	12	6.5	3.5
JHS	369	47.4	0.0	12	4.7	3.3
SHS	369	1.3	0.0	7	0.8	1.1

What difficulties are experienced by pre-service teachers when solving non-routine mathematical problems?

The results from the data collected on trainees' difficulties through the interviews and analysis of their worksheets are presented in this section. Table 3 presents a summary of trainees' difficulties in solving the non-routine word problems.

Table 3
Category of Difficulty of Pre-Service Teacher Trainees

Category	P1	P2	P3	P4	P5	P6
Reading	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Comprehension	6 (33.3)	14 (77.8)	3 (16.7)	2 (11.1)	0 (0)	9 (50)
Transformation	8 (44.1)	15 (83.3)	5 (27.8)	2 (11.1)	7 (38.9)	11 (61.1)
Processing	13 (72.2)	18 (100)	10 (55.6)	2 (11.1)	12 (66.7)	14 (77.7)
Encoding	NO	NO	NO	NO	NO	NO

Note: (1) the values in the parentheses represent percentages.
(2) P1-Problem 1, P2 – Problem 2, P3 – Problem 3,

P4 – Problem 4, P5 – Problem 5, P6 – Problem 6.
(3) NO – Not Observed during the interviews.

The results show that Problem 2 was the most difficult for the participants who were interviewed, while Problem 4 was the easiest. None of the participants had difficulty reading the problems. However, understanding of some of the problems was difficult for quite a number of the trainees. The majority of them had difficulty understanding Problem 2 (77.7%). A half (50%) had difficulty understanding the demands of Problem 6. While a third (33.3%) had difficulty understanding Problem 1. Problem 5 was the only problem that all the trainees were able to comprehend (see Appendix A). Excerpts from the interviews with a trainee from College C below show the difficulty some of the trainees had understanding Problem 3, for example, because of too many words:

R: Kindly read the problem.
SC9: [Reads the question fluently without any problem].
R: Do you understand the problem?
SC9: No.
R: Why?
SC9: It is very confusing.
R: Why is it confusing?
SC9: there are too many sentences.

Transformation of the problems correctly into mathematical equations was very difficult for the majority of the trainees. In solving Problem 6, Participant SC25, for example, was able to transform the first part of the problem but had extreme difficulty transforming the second part of the problem, as shown in Figure 2. Participant SC25 had difficulty transforming "3 less than answer" into mathematical sentence, hence she introduced an inequality sign because of the "less than" and therefore presented the solution as shown in Figure 2, instead of transforming the problem into the equation: $\frac{x-3}{5} = \frac{x-5}{3} - 3$ or $\frac{x-3}{5} + 3 = \frac{x-5}{3}$ and proceed with the solution as shown in Appendix B.

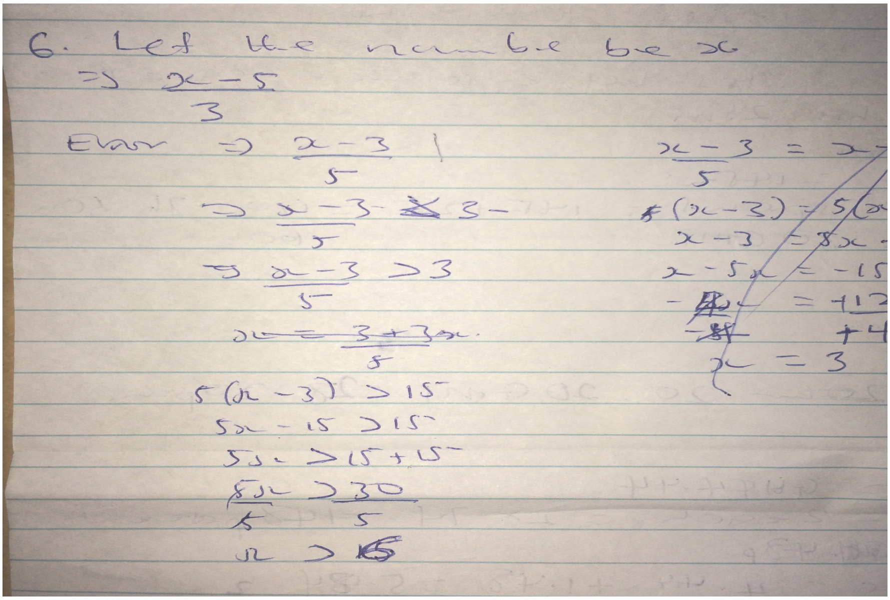

Figure 2. Participant SC25's solution to Problem 6.

Participant SB4 also had difficulty transforming Problem 2. She identified information given in the problem and set up the solution path by first transforming the problem algebraically. She added the cost of the hat to that of the shirt and belt and set up an equation using *x* as the unknown amount of money in the boy's wallet. She divided the resulting amount (which was *GH¢30.00*) by three, the number of items the boy bought, and obtained *GH¢10.00* as the starting amount as shown in Figure 3, instead of *GH¢17.50* (see Appendix B).

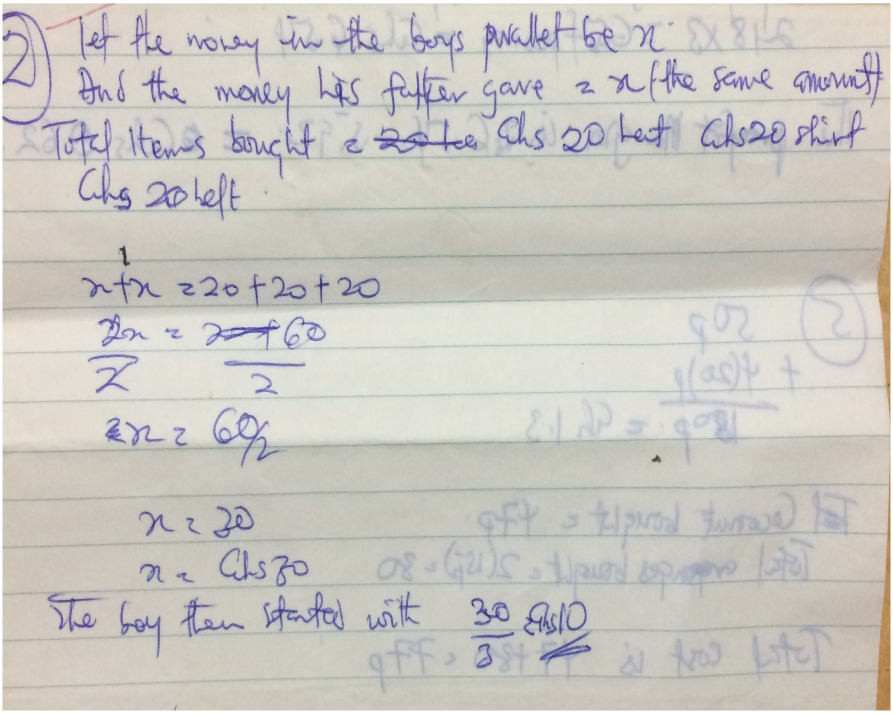

Figure 3. Participant SB4's solution to Problem 2

Processing of most (5 out of 6) of the problems was very difficult for the trainees. With the exception of Problem 4 in which the majority (88.2%) of the trainees were able to process, processing of the rest of the problems was very difficult for the trainees. Problem 4 was the only problem in which the same number of trainees who had difficulty with comprehension of the problem also had difficulty with the process. For the rest of the problems, the number of trainees who had difficulty with processing were higher than the number who had difficulty with comprehension. For example, in Problem 5 where all the trainees exhibited very good understanding of what the problem required them to do, the majority (66.7%) of them had difficulty processing it. This shows that even trainees who understood the problem had difficulty marshalling resources including heuristics in problem solving to unlock the problem.

In solving Problem 3, for example, Participant SB120 from College B read the question and explained the demands of the question without problems but in processing the question had the wrong answer because of careless error and lack of application of some useful heuristics in problem solving such as looking back, as shown in his explanation below:

> First, take 2 hours 25 minutes and add 10 minutes. It gives 2 hours 35 minutes. Then 10 hours and 30 minutes minus 2 hours and 35 minutes. 10 hours minus 2 hours equal 8 hours, and 35 minutes minus 30 minutes is equal to 5 minutes. Answer is 8 hours 5 minutes. To minus 15 minutes, I take 8 hours 5 minutes minus 15 minutes. I cannot minus, therefore I change 8 hours to 7 hours and 5 minutes become 95 minutes. I have 8 hours and 5minutes which give 8.05pm.

10 hours 30 minutes take away 2 hours 35 minutes will not give 8 hours 5 minutes but rather 7 hours 55 minutes. This careless error resulted in the wrong answer of 8.05pm instead of 7.40 pm, that is, 7 hours 55 minutes take away 15 minutes. Unfortunately SB10 did not look back to check the answer.

In solving Problem 4, Participant SA5 also had the final answer wrong because of careless error and failure to look back as shown in the excerpt of the interview below:

> I found the quantity of the sour oranges by dividing the quantity of the sweet ones by two. Afterwards I solved for the cost price of the sour oranges as (2p × 74) = GH¢14.8. I then sum up the two cost prices for the sour and sweet oranges and had GH¢20.24. Now I sum up the cost price of the oranges as GH¢22.2 then I subtracted the selling price from the cost price to get GH¢1.96. Now, I found the percentage profit as 1.96 divided by 22.2 and then multiply (sic) the result by 100 per cent to get 8.28 per cent.

Participant SA5's explanation shows that she understood the problem and knew how to solve it but failed to get the correct answer because of careless error and failure to look back.

Others who showed good understanding of the problem also had difficulty processing the question because they had no clear idea how to proceed with the solution process as shown in the example in Figures 4 and 5.

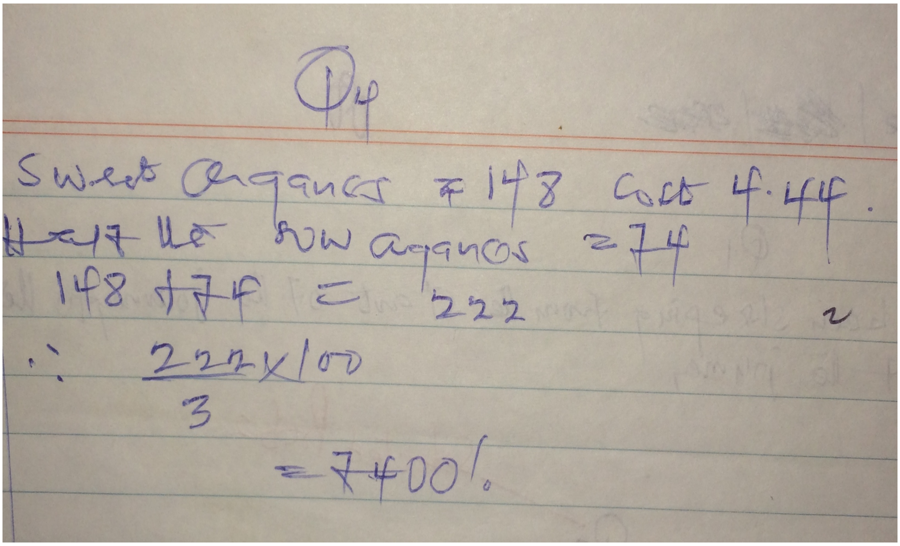

Figure 4. Participant SA17’s solution to Problem 4

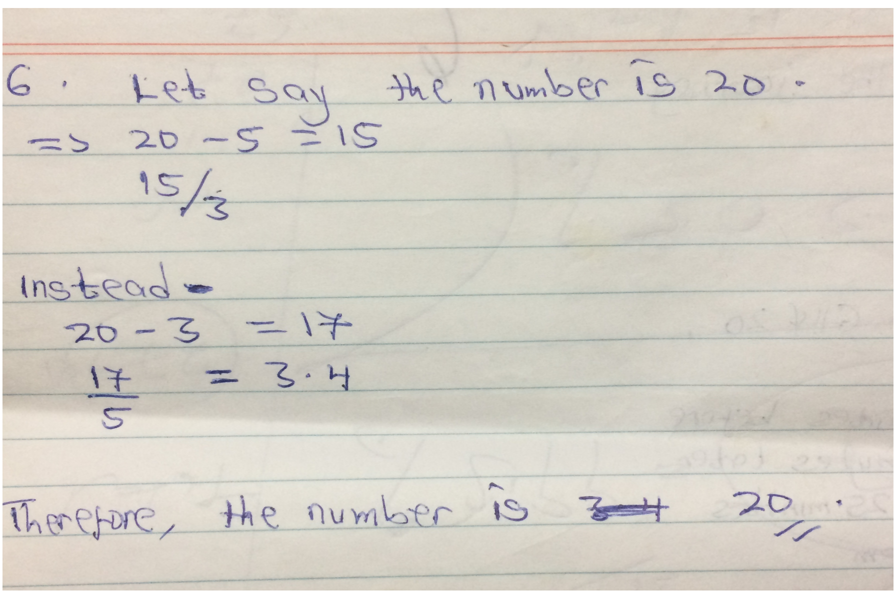

Figure 5. Participant SA53’s solution to problem 6.

The strategy most of the trainees used in processing the problem was guess followed by number manipulation. They however failed to look back to check the reasonableness of their answers, hence they were unsuccessful in most cases. Figure 5, for example, shows how Participant SA3 solved Problem 6 using guesses and number manipulation. In order to get a sense of the unknown number, Participant SA3 used the values 3 and 5 in the question and guessed the value of the unknown to be 20. Not being so sure of his solution, he presented two sets of solutions as shown in Figure 6. In one case, he subtracted 5 from the number he guessed (20) and then divided the result by 3. In the other instance, he subtracted 3 from the 20 and then divided the result by 5. He did not look back to check whether the solution was right or wrong.

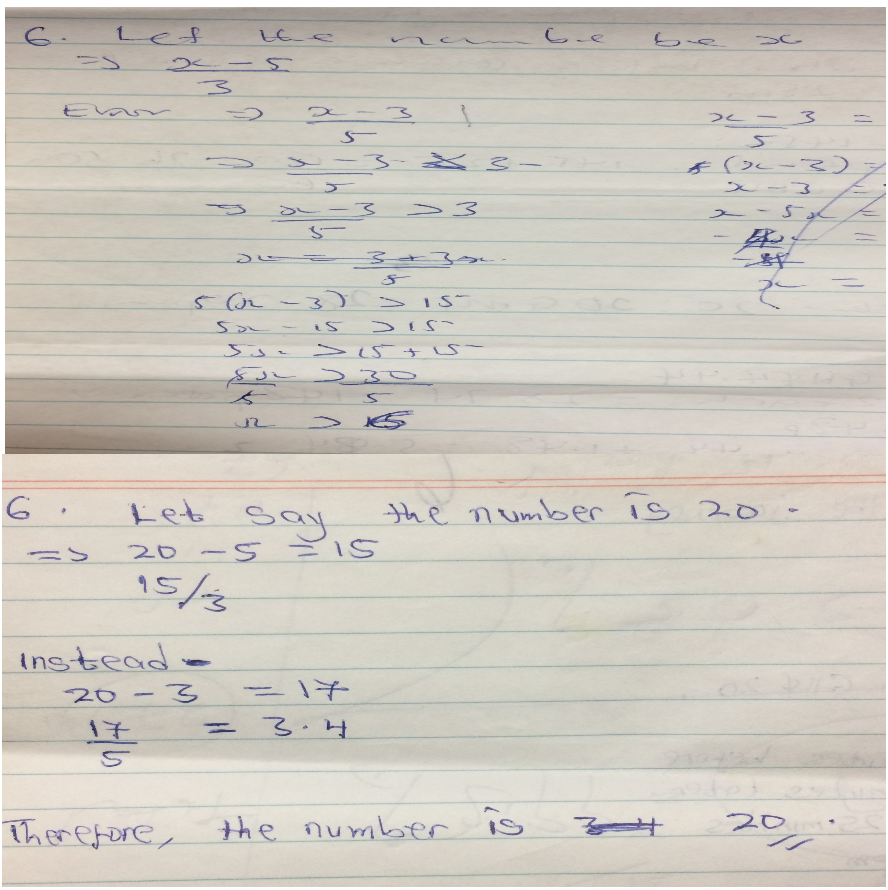

Figure 6. Participant SA3's answer to Problem 6

In solving Problem 5, Participant SC92 also employed guess and number manipulation but did not check the reasonableness of her answer as shown in the explanation below:

> Firstly, I found the expenditure of Mr. Owusu as forty-seven pesewas plus fifteen pesewas and had sixty-two pesewas. Since he had a fifty pesewas coin and four of the twenty pesewas coin, he will use the fifty pesewas coin to buy the coconut, he will be given a change of three pesewas and then use one of the twenty pesewas coin to buy the orange and had a change of five pesewas. Next, I sum up the three pesewas and the five pesewas and had eight pesewas. Therefore, the amount left on him after his expenditure is eight pesewas plus three times the twenty pesewas which has not been used to obtain sixty-eight pesewas.

Participant SC92 guessed that Mr. Owusu used the fifty pesewas coin to buy the coconut but ended up with the wrong solution due to careless error. Like the other participants, SC92 never looked back to check the reasonableness of the answer.

The researchers did not observe encoding of the problem during the interviews. None of the trainees looked back to ensure that their solutions addressed each the problems they had set out to solve. Some trainees just used the four basic operations to combine the values given in the problem without looking back to check whether they had addressed the demands of the problem, as shown in Figure 6, for example.

Conclusions and Implications

A number of conclusions were made from the results of this study. First, the results showed that the teacher trainees performed poorly on the test. For instance, while 58.9% of participants passed on the Primary category of questions (which implies that about two-fifths of them could not pass the non-routine word problems at the primary school level), more than a half (52.6%) failed the items based on the JHS level content. In addition, the overwhelming majority (98.7%) failed the items based on the SHS content. Granted that these trainees had gone through Senior High School Education and passed all examinations and have also gone through two years of content training in their teacher training programme (IoE, 2005, 2014), one would have expected them to perform better than they did especially on the SHS level items, where the pass rate was less than 2%. Thus, participants' ability to solve the mathematical tasks declined in the JHS and SHS category of problems, with almost all of them failing the task at the Senior High School level. This finding seems to confirm literature on the weak content knowledge on prospective elementary school teachers (Ball, Hill & Bass, 2005; Tirosh, 2000). One implication of this is that one cannot be sure how well such participants, after their programmes, could effectively teach content at the JHS level. Also, since literature suggests direct relationship between proficiency in problem solving and capacity to pose problems (Rosli, Capraro, Goldsby, Gonzalez, Onwuegbuzie & Capraro, 2015), another implication of such low performance of participants is that by not being good problem solvers they are more likely not to have the ability to pose good problems as teachers; an ingredient necessary for effective teaching.

Second, while all the trainees were able to read all the problems without any difficulty, quite a number of them had difficulty understanding about half of the problems. As has already been discussed, Problem 2 was the most difficult for the majority of the trainees to understand (88.2%). This was followed by Problem 6 (64.7%) and Problem 1 (47.1%). Literature suggest that linguistic-related difficulties associated with issues such as syntax, and lack of understanding of mathematics register may contribute to this (Spanos, Rhodes, Dale & Crandall, 1988; Daroczy, Wolska, Meurers and Nuerk, 2015). The present study did not investigate such issues. Further studies are therefore recommended to help unearth why participants similar to those used in the present study could read the problems/tasks presented to them fluently but still have difficulty in comprehending the demands of the tasks.

Third, participants were stuck in most cases because of difficulties they had in transforming the non-routine word problems into correct mathematical equations as shown in Figure 2 and Figure 3, for example. Consequently, processing of the problems including those the participants understood was difficult for them. This finding is similar to that of Rosli, Capraro et al. (2015), who also found that majority of preservice teachers in their study who understood a problem statement had difficulty transforming it into an equation. In some cases in this study, participants just guessed the unknown and manipulated the numbers provided in the questions meaninglessly with the hope of finding the solution, without looking back to check the reasonableness of their answers. Participants were also often held back due to factors such as their inability to analyse the mathematical structure of the problem, especially relationships amongst various quantities as was seen in Figure 4. Schoenfeld (1985) identified heuristics as an important factor in problem solving. By not being able to transform the non-routine word problems into the appropriate mathematical equations or relations, participants in this study could not apply appropriate heuristics needed to solve the problems. As a result, careless computations were identified as a challenge, a pattern similar to what was reported in Newman's (1983) study. Our own read of the College of Education curriculum revealed that while conscious efforts have been made to provide training in the area of mathematics content, methods of teaching mathematics and the study of the Primary and JHS curricula in the training programme, not much attention is paid to training in the area of mathematics problem solving generally, and non-routine word problem solving particularly (IoE, 2005, 2014). This is likely to affect the teaching and learning of non-routine mathematics problem solving in primary school since literature suggests that teachers' knowledge affect students' opportunities to learn mathematics generally (Huckstep, Rowland & Thwaites, 2003). In sum, this finding points to the need for explicit instruction of various heuristics necessary for solving non-routine mathematics problems at the Primary and JHS levels to teacher trainees in Ghana. Literature suggests that training in problem solving, especially in strategy and heuristics enhance the problem-solving capability of the learners (Schoenfeld, 1985; Yeo, 2009). Therefore, at the College of Education level, it will be necessary to develop a course in mathematics problem solving for trainees to equip them with the knowledge and skills they need to be effective problem solvers. Professional development courses in the area are also recommended for the already practicing teachers since they may also have such deficiencies.

Finally, although this study was carried out in only 3 (out of 40) public Colleges of Education in Ghana, the findings may shed light on what might be happening in the other Colleges of Education in the whole country and other developing countries that share similar situation as Ghana. This implies that there is the need for a large scale research to ascertain the proficiency of teacher trainees at the Colleges of education in non-routine algebraic word problems, especially at the Junior High School level.

References

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 14-17, 20-22, & 43-46.*

Brews, A. (2001). *The Nature of research inquiry in academic contexts.* New York: Routledge Falmer.

Creswell, J. W. (2012). *Educational research: Planning, conducting and evaluating quantitative and qualitative research (4th ed.).* San Francisco, CA: Jossey-Bass.

Daroczy, G., Wolska, M., Meurers, W. D., & Nuerk, H.-C. (2015). Word problems: a review of linguistic and numerical factors contributing to their difficulty. *Frontiers in Psychology, 6*(348), 1–13.

Fraenkel, J. R., & Wallen, N.E. (2006). *How to design and evaluate research in education* (3rd Edition.). New York: McGraw-Hill.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Diana, W., Murray, H., Olivier, A., & Human, P. (1997). *Making sense teaching and learning mathematics with understanding*. Partmouth, NH: Heinemann.

Huckstep, P., Rowland, T., & Thwaites, A. (2003). *Primary teachers' mathematics content knowledge: What does it look like in the mathematics classroom?* Retrieved from www.leads.ac.uk/educol/documents/00002534.htm.

Intsiful E. C. (2014). *Linguistic influences on JHS students' performance on mathematics problem solving* (Unpublished master's thesis). University of Cape Coast, Cape Coast.

IoE. (2005). *Institute of Education, University of Cape Coast, three-year diploma in basic education, course structure (revised syllabus).* Institute of Education, University of Cape Coast, Cape Coast.

IoE. (2014). *University of Cape Coast, College of Education Studies, Institute of Education, draft three-year diploma in basic education, five-semester programme (revised syllabus)*. Institute of Education, University of Cape Coast, Cape Coast.

Kilpatrick, J. (1985). A retrospective account of the past twenty-five years of research on teaching mathematical problem solving. In Silver E. A. (Ed). *Teaching and learning problem solving: Multiple research perspectives*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Kinach, B. (2002). Understanding and learning-to-explain by representing mathematics, *Journal of Mathematics Teacher Education*, *5*, 153-186.

Lappan, G., & Briars, D. (1995). How should mathematics be taught? In I. M. Carl (Ed.), *Prospects for school mathematics* (pp. 131-156). Reston, VA: National Council of Teachers of Mathematics.

Mertens, D. M. (2003). Mixed methods and the politics of human research: The transformative - emancipatory perspective. In: A. Tashakkori & C. Teddlie (Eds.), *Handbook on mixed methods in the behavioural and social sciences* (pp. 135-164). Thousand Oaks, CA: Sage.

Ministry of Education (2010). *Teaching syllabus for mathematics (senior high school 1 - 3)*. Accra: Curriculum Research and Development Division, GES.

Ministry of Education (2012a). *Teaching syllabus for mathematics (primary school 1 - 6)*. Accra: Curriculum Research and Development Division, GES.

Ministry of Education (2012b). *Teaching syllabus for mathematics (junior high school 1-3)*. Accra: Curriculum Research and Development Division, GES.

Mullis, I. V. S, Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre.

National Council of Teachers of Mathematics, NCTM. (2000) *Principles and Standards for School Mathematics*, Reston, Virginia: National Council of Teachers of Mathematics. Netherlands: State University of Utrecht.

Newman, M. (1977). An analysis of sixth-grade students' errors on written mathematical tasks. In M. A. Clements & J. Foyster (Eds.), *Research in Mathematics Education in Australia, 1977* (vol. 1, pp. 239-258). Melbourne: Swinburne Press.

Newman, M. A. (1983). *Strategies for diagnosis and remediation.* Sydney: Harcourt, Brace Jovanovich.

Osiakwan, J. K., Wilmot, E. M., & Sokpe, Y. B. (2014). Relationship between teachers' knowledge for teaching and students' achievement in algebra in some selected senor high schools in Ghana. *Elixir Educational Technology*, *75*, 27782-27787.

Person, A.C., Berenson, S.B., & Greenspan, P.J. (2004). The role of number in proportional reasoning: A prospective teacher understanding. In M. Hoines & A. Fuglestad (Eds), *Proceedings of PME 28* (pp. 17-24). Bergen, Norway: PME.

Polya, G. (1973). *How to solve it*. Princeton, NJ: Princeton University Press.

Posamentier, A. S., & Krulik, S. (2008). *Problem-solving strategies for efficient and elegant N solutions grades 6-12.* Thousand Oakes, CA: Corwin Press.

Reed, S. K. (1999). *Word problems research and curriculum reform*. NJ: Lawrence Erlbaum.

Rosli, R., Capraro, M.M., Goldsby, D., Gonzalez, E, G. Y, Onwuegbuzie, A.J. & Capraro, R. M. (2015). Middle-grade preservice teachers' mathematics problem solving and problem posing. In F. M. Singer, N. F. Elleeron & J. Cai. (Ed). *Mathematical problem posing from research to effective practice* (pp. 333-354). NY: Springer.

Sánchez, V., & Llinares, S. (2003). Four Student Teachers' pedagogical reasoning on functions. *Journal of Mathematics Teachers Education*, *6*(1) 5-25.

Schoenfeld, A. H. (1985). *Mathematical problem-solving*. Orlando: Academic Press.

Spanos, G., Rhodes, N. C., Dale, T. C., & Crandall, J. (1988). Linguistic features of mathematical problem solving. In R. R. Cocking & J. P. Mestre (Eds.), *Linguistic and cultural influences on learning mathematics* (pp. 221-240). Hillsdale, NJ: Lawrence Erlbaum Associates.

Taplin, M. (1995). Mathematics remediation requirements for pre-service teachers. *New Horizons in Education, 92*, 3-19.

Tirosh, D., & Graeber, A., (1989). Pre-service elementary teachers' explicit beliefs about multiplication and division. *Educational Studies in Mathematics, 20,* 79-96.

Tirosh, D., Tirosh, C., Graeber, A., & Wilson, J. (1991). Computer-based intervention to correct pre-service teachers' misconceptions about the operation of division. *Journal of Computers in Mathematics and Science Teaching, 10,* 71-78.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal of Mathematics Teacher Education, 31*, 5-26.

Torner, G., Schoenfeld, A. H., & Reiss, K. M. (Eds.) (2007). Problem solving around the world: summing up the state of the art. *ZDM Mathematics Education, 39*(5/6), 353–551.

UNESCO. (September, 2010). *World data on education, 7th edition, 2010/2011.* Retrieved from www.ibe.unesco.org/en/document /world-dataeducation-seventh-edition-2010-11.

Van Doormen, W., Verschaffel, L., & Onghena, P. (2003). Pre-service teachers' preferred strategies for solving arithmetic and algebra word problems. *Journal of Mathematics Teacher Education*, *6*(1), 27-52.

Wilmot, E. M. (2009). Teacher knowledge and student performance: Begle re-visited in Ghana. *Journal of Science and Mathematics Education*, *4*(1), 13-30.

Yeo, K. K. J. (2009) Secondary 2 students' difficulties in solving non-routine problems. *International Journal for Mathematics Teaching and Learning*, *8*, 1-30.

Appendix A

Non-Routine Problem-solving Test for Level 200 DBE Students

Student ID Number: …………
Sex: …
Age: ….
College: ………..
Programme of Study: ……

This is a non-evaluative assessment. Your performance in this exercise will have no effect on your final grade or continuous assessment mark in your course. The assessment is designed to elicit information that will help in understanding how you carry out mathematics problem solving.

<u>Answer all questions.</u>
<u>Show all working clearly on the answer booklet.</u>
***Time allowed 1hour*.**

1. A passenger who had travelled half of his journey fell asleep. When he awoke, he still had to travel half the distance that he had travelled while sleeping. For what part of the entire journey had he been sleeping?
2. A boy went shopping with his father. He found a hat he wanted to buy for *GH¢20*. He said to his father, "If you will lend me as much money as I have in my wallet, I will buy the hat." His father agreed. They then did it again with *GH¢20* shirt and with a belt *GH¢20*. The boy was finally out of money. How much had he started with?
3. Miss Konadu arrived at the concert hall 15 minutes before a concert began. However, due to some technical problems, the concert started 10 minutes later. The whole concert lasted for 2hours 25 minutes. It was 10.30 pm when Miss Konadu left the concert hall. At what time did Miss Konadu arrive at the concert hall?
4. A market woman bought a basketful of 148 sweet oranges for Ghs 4.44. She later bought half the same quantity of sour oranges at 2pesewas each. She mixed them and sold them at 3pesewas per orange. What is her gain or loss in percentage?

5. Owusu has one 50p coin and four 20p coins. He buys a coconut for 47p and 2 oranges for 15p each. How much money does he have left?
6. A boy was asked to subtract 5 from a certain number and divide the result by 3. Instead he subtracted 3 from the number and divided the result by 5. His answer was 3 less than it should have been. Find the number.

Appendix B

SOLUTIONS TO THE NON-ROUTINE PROBLEM-SOLVING TEST FOR LEVEL 200 DBE STUDENTS

<u>Solution to Problem One</u>

Let x be the fraction of the total distance during which he has to sleep. Then when he awoke he had a distance of $\frac{1}{2}x$ left to travel. Thus the distance from when he fell asleep to the end of the journey was $x + \frac{1}{2}x = \frac{3}{2}x$. This is one-half of the total distance. Thus we have:

$$x + \frac{1}{2}x = \frac{1}{2}$$
$$\frac{3}{2}x = \frac{1}{2}$$
$$x = \frac{2}{3} \times \frac{1}{2}$$
$$x = \frac{1}{3}$$

Thus he slept for one-third of the journey.

<u>Solution to Problem Two</u>

Let x be the amount of money (in GH¢) that the boy started with. Then his father lent him more Ghana cedis so that he had a total of 2 Ghana cedis.

Of this amount, he spent GH¢ 20 on the hat leaving him with:

$2x - 20$

His father then lent him as much as he already had so that he then had: $2(2x - 20)$

He then bought the GH¢ 20 shirts, leaving him with:

$2(2x - 20) - 20$

His father doubled the above, leaving him with:

$$2[2(2x - 20) - 20]$$

He spent GH¢ 20 more and had no money left. Thus we have:

$$2[2(2x - 20) - 20] - 20$$
$$8x - 140 = 0$$
$$8x = 140$$
$$x = 17.5$$

Thus the boy started out with GH¢ 17.5

<u>Solution to Problem Three</u>

Miss Konadu left the concert hall at 10.30 pm

If the concert lasted for 2 hours 25 minutes.

Then: 10hours 30 minutes 2 hours 25 minutes 8 hours 5 minutes

Thus the concert started at 8.05 pm.

But the concert delayed and started *10minutes* later at this time *(8.05 pm)*, therefore the time of start of the concert should have been: *8hours 5 minutes – 10 minutes = 7 hours 55 minutes.*

Thus the concert should have started originally at exactly*7.55 pm.* Now, if Miss Konadu entered the concert hall *15minutes* before the original time of start of the concert (7.55 pm), then she arrived at the concert hall at: 7 hours 55 minutes − 15 minutes = 7 hours 40 minutes.

Thus Miss Konadu arrived at the concert hall at*7.40 pm.*

Solution to Problem Four

Basketful of 148 sweet oranges = GH¢ *4.44*

Sour oranges = Half of basketful of sweet oranges = *74*

If one sour orange cost 2pesewas then 74 will cost her: $74\times 2 = 148$ *pesewas or* GH¢ *1.48*

Therefore, total cost of purchase of oranges = $4.44 + 1.48$ = GH¢ *5.92*

If she mixed them, then she has a total of: $148 + 74 = 222$ oranges to sell at *3pesewas each*.

Then she made total sales of: *222×3=666pesewas or* GH¢ *6.66*

Gain/profit = $6.66 - 5.92$ = GH¢ *0.74 or 74pesewas*

Gain per cent = *12.5%*

Solution to Problem Five

Owusu had a total of $[50 + 4(20)]\ p = [50 + 80]\ p = 130p$

Amount spent on coconut and oranges = $47p + 2(15)\ p = 47 + 30 = 77p$

Therefore he has $(130 - 77)\ p = 53p$ *left*.

Solution to Problem Six

Let x be the number, then**;** $(x - 5) \div 3 = [(x - 3) \div 5] + 3$

The LCM of 3 and 5 = 15

$\therefore\ 5(x - 5) = 3(x - 3) + 45$

$5x - 3x = 45 + 25 - 9$

$2x = 61$

$x = 30.5$