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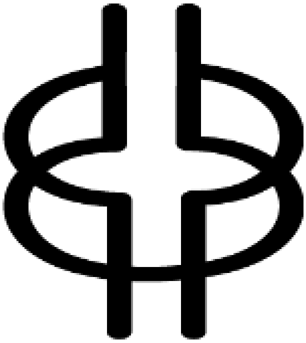

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Reconceptualising Teacher Knowledge in Domain Specific Terms

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Abstract

Literature is replete with different conceptualizations of teacher knowledge, each with their implications on teaching practices. Pivotal among these are conceptualizations of content knowledge, curriculum knowledge, pedagogical knowledge and pedagogical content knowledge. The paper argues that until now, the various conceptualizations of teacher knowledge have been mostly general and not domain specific enough. In addition, researchers who have relied on these earlier conceptualizations have mainly concerned themselves with teacher knowledge qualitatively. In an attempt to shift from these general conceptualizations and the qualitative measures of teacher knowledge, the study on which this paper is based relied on the Knowledge of Algebra for Teaching (KAT) project's conceptualization of teacher knowledge for teaching algebra and an adaptation of one of the instruments. Two hundred and nine participants comprising 189 prospective and 20 in-service high school mathematics teachers in Ghana took part in the study. From the findings of the study, this paper argues that effective mathematics teachers do not only use the individual conceptualized knowledge but sometimes also blend these types of knowledge into somewhat new forms of knowledge. Consequently, the paper proposes an elaboration of the KAT project's domain specific conceptualization of teacher knowledge that recognizes overlapping packages of knowledge and lends itself to being assessed both qualitatively and quantitatively.

Key words: Knowledge for teaching algebra, pedagogical content knowledge in algebra, advanced algebra teaching knowledge, school algebra teaching knowledge.

Introduction

For many years, researchers in the field of education have expressed divergent opinions about which school factors influence student achievement. According to Duthilleul and Allen (2005), this debate was started in the US after Coleman et al., (1966) analyzed data from about 600,000 students and 60,000 teachers in more than 4,000 schools and, in their report entitled Equality of Educational Opportunity, concluded that only about 10 percent of the variance in student achievement could be explained by school factors. In that report, they stated "that family background characteristics and community level variables accounted for more variance in student achievement than school resource variables like . . . teacher characteristics" (p.3). This finding led many people to even question whether schools matter at all in student learning.

In the intense debate that ensued thereafter, viewpoints emerged that questioned whether schools matter in student learning. Wilmot (2008) has argued that, "such negative findings and views about schools and teachers in particular could be the impetus of early attempts at conceptualizing the knowledge base for teaching that was spearheaded by Shulman (1986a, 1986b, and 1987)" (p. 33). The point needs to be made that initial attempts at conceptualizing the knowledge for teaching were not only aimed at debunking the aforementioned conclusion made by the Coleman committee but also aimed at establishing the idea that a specialized type of knowledge is needed to become a successful teacher, thereby improving the value and image of teachers. This was the issue Strom (1991) aptly made when he said, "at one level, concern about the knowledge base focuses on improving the respect and status accorded teaching, thereby making it a more rewarding career" (p. 1). Strom (1991) was indirectly arguing that making a case that teaching involved a wise application of a specialized body of knowledge was necessary to also emphasize the point it (teaching) influences learning outcomes.

It can be argued that the idea of *pedagogical content knowledge* (PCK) introduced by Shulman (1986b) was quite pivotal in the wave of research into teacher knowledge. Not only that, the conceptualizations by Shulman and his colleagues of *content knowledge* and *pedagogical content knowledge* and the distinction between them threw the brightest light on how teacher knowledge could influence teaching and brought the attention of researchers in several content domains to issues involving the type of knowledge teachers need about content for teaching, different from what an ordinary adult may have (see for example, Ball, 1988; Wilson & Winneburg, 1988; Grossman, 1990). It can even be argued that the conceptualization of *technology pedagogical content knowledge* (TPACK) (see Koehler & Mishra, 2008 Mishra, Koehler, & Henriksen, 2011) is partly influenced by the PCK conceptualization earlier formulated by Shulman and his colleagues.

Though the work by Shulman and his colleagues threw the brightest light on issues of teacher knowledge, prior to their conceptualization earlier researchers had proposed several other conceptualizations (see reviews done by Doyle, 1977; Gage, 1978; Brophy & Good, 1986, as well as studies such as, Berliner, 1979; Peterson & Swing, 1982; Leinhardt & Smith, 1985). For instance, Leinhardt and Smith (1985) proposed *lesson structure knowledge* (LSK) and *subject matter knowledge* (SMK) as two types of teacher knowledge. According to them, LSK comprises smooth planning and organizing of lessons and providing clear explanations, while SMK consists of concepts, algorithmic operations, connections among different algorithms and knowledge of the types of errors students make.

The point also needs to be made that another effect of the 1966 Equality of Educational Opportunity report by Coleman and his colleagues was the interest it generated, and rightly so on investigations into the effects of instruction on student learning. By the mid 1980s enough studies had established the fact that schools in general and teachers in particular matter in student learning. Presently there is agreement, among a number of researchers that one of the most important factors affecting student learning outcomes is the teacher (see for example, Jordan, Sanders & Rivers, 1996; Mendro, & Weerasinghe, 1997; Wright, Horn, & Sanders, 1997; Wilmot, 2009). Wright and his colleagues, for instance, concluded that “Effective teachers appear to be effective with students of all achievement levels, regardless of the level of heterogeneity in their classrooms. If the teacher is ineffective, students under the teacher's tutelage will show inadequate progress academically despite how similar or different they are regarding their academic achievement (Wright et al., 1997, p. 63). And as if to tie such efforts up together, Brophy and Good (1986) reviewed a number of studies conducted since the 1970s and concluded that, “The myth that teachers do not make a difference in student learning has been refuted” (p. 370).

In spite of the refutation of this myth, researchers have continued on investigations towards re-conceptualization of the knowledge base for teaching different from Shulman's framework in more domain specific terms. Notable among these is the work by Liping Ma. Ma (1991) undertook a study in which she interviewed 95 elementary schoolteachers, 72 from China and 23 from the U.S. In *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States,* Ma (1999) discusses the outcomes of this study from which she introduced a different kind of conceptualization of the knowledge base for teaching, which she termed *Profound Understanding of Fundamental Mathematics* (PUFM). It can be argued that Ma's conceptualization is different from that of Shulman in that while Shulman's (1986b) conceptualization is a generalized form of knowledge that is essentially not domain specific, Ma's (1999) conceptualization is limited to mathematics (i.e., Ma's is more domain specific than Shulman's). However, the two conceptualizations show some semblance in the sense that both involve a complex combination of some form of content and pedagogical knowledge except, as already pointed out, that whereas Ma's conceptualization is restricted to the subject matter of mathematics, Shulman's seem to be related to a generalized content, not necessarily mathematics.

All the aforementioned studies produced mostly qualitative information about teachers' knowledge and its influence on their teaching practice. In a subject, such as mathematics, though such qualitative information are worthy, quantitative measures are also necessary to ensure that mathematics teachers have a good knowledge of the mathematics students are required to learn in school. As a result, a number of proxy measures have been used to measure teachers' knowledge of the content of school mathematics, as well as issues related to pedagogy. In the US, for instance, to be certified to teach mathematics, various states have taken steps in this direction by requiring pre-service teachers to pass a mathematics test prior to being certified to teach. An example of this type of test is PRAXIS, a widely used teachers' licensing examination developed by Educational Testing Service (ETS). In spite of these steps, the quality of achievement of K-12 students in mathematics continues to be of national concern. Consequently, the RAND Mathematics Study Panel (2003) made a number of recommendations for improving teachers' mathematical knowledge for teaching. These include the need for further clarification of the knowledge demands of teaching mathematics, and a deeper understanding of ways to provide opportunities for prospective and practicing teachers to acquire this kind of knowledge. In addition, the RAND Mathematics Study Panel (2003) recommended the development of instruments for assessing the mathematical knowledge for teaching across grade levels and mathematical domains. The RAND panel also singled out algebra as an important area of focus in all these efforts.

As if to respond to the recommendations by the RAND panel, within the last one and half to two decades two major studies have focused not only on reconceptualizing the knowledge base for teaching mathematics in the U.S. but also developed instruments to measure it. These are the works by Deborah Ball and her colleagues at the elementary school level (Ball & Bass, 2000; Hill, Schilling & Ball, 2004; Hill, Rowan & Ball, 2005) and Ferrini-Mundy and her colleagues on the Knowledge of Algebra for Teaching (KAT) project at the high school level (see for instance, Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, Senk, & McCrory, 2005; Ferrini-Mundy, Senk, McCrory, & Marcus, 2005).

At the elementary school level Deborah Ball and her colleagues' work on elementary school mathematics teachers' knowledge introduced the idea of mathematical knowledge for teaching. Ball and her colleagues relied on existing theories about teacher knowledge and developed survey-based questions based on teaching mathematics at the elementary school level. Through factor analyses of the data they obtained from the administration of their questions, they proposed, among other things, the idea of "specialized knowledge of content" (SKC). As they put it,

> In addition to a general factor, specific factors representing knowledge of content in number and operations, knowledge of students. . . [there is also] a specialized knowledge of content (SKC) made up of several items: representing numbers and operations, analyzing unusual procedures or algorithms and providing explanations for rules. (Hill, Schilling & Ball, 2004, pp 27-28).

At the high school level, the work by Ferrini-Mundy and her colleagues on the KAT project was not only aimed at reconceptualizing teacher knowledge in a domain specific manner but also as a measurable construct. As earlier mentioned to the extent that this project focused on Algebra among all the domains of mathematics the KAT project could be said to have been influenced by the recommendations from the RAND Mathematics Study Panel (2003). Their work also appears to be groundbreaking at the high school level as it did not only end with reconceptualization of knowledge of mathematics for teaching algebra but also developed measures of it.

It is necessary to state that the work by Ferrini-Mundy and her colleagues on the KAT project is not only important in reconceptualization and measurement of teacher knowledge in algebra, it can also be applied to answer the question of which aspects of teacher knowledge best relates to student performance for researchers interested in linking teacher knowledge with student performance. When earlier attempts were made to link teacher knowledge to student performance it was mostly limited to teacher actions design (see for example, Berliner, 1979; Peterson & Swing, 1982). For instance, Berliner and his colleagues in the Beginning Teacher Evaluation Study (BTES) introduced a variable, which they called *Academic Learning Time* (ALT) in their modification of the process-product research design and insisted among other things that this variable serves as the link between teacher behaviour and student achievement. Unfortunately, by restricting the focus on teacher actions the BTES program failed to show what type of knowledge teachers use in judging the difficulty level of the tasks they give to their students, especially in heterogeneous situations where students come with varied backgrounds. In addition, their program could not show which aspect of teacher knowledge affects student performance.

The reconceptualization by the KAT project and the development of instruments to measure teacher knowledge attempts to fill this gap in that it opens the door, as it were, for researchers to rely on analyses such as multiple linear regression and structural equation modeling to determine which aspect of teacher knowledge affects student performance in any circumstances. It is hoped that successful validation or expansion of the conceptualization of the KAT hypotheses could lay the foundation for studies aimed at answering the question as to which aspect of teacher knowledge best relates to student performance. This is because such a validated framework or new conceptualization would lend itself to being assessed both qualitatively and quantitatively and on a large scale and this way have implications for teacher educators. It is in the light of this that this study was conducted to attempt to validate the KAT framework, where possible, or improve it to make it more applicable to other domains of learning.

Conceptual Framework

Through analyses of research literature, recommendations by professional organizations and videos of teaching, researchers in the Knowledge of Algebra for Teaching (KAT) have hypothesized that the knowledge used by teachers in teaching school algebra consists of three types. These are *knowledge of school algebra* (referred to in short as *school knowledge*), *advanced knowledge of mathematics* (also referred to as *advanced knowledge*), and *teaching knowledge*. These three types of knowledge, discussed in turn below, constitute the theoretical frame of algebra knowledge for teaching that guided this study.

Knowledge of School Algebra/School Knowledge

According to researchers of the Knowledge of Algebra for Teaching project, the first category of knowledge, *Knowledge of School Algebra* (or simply *School Knowledge*), is the knowledge of mathematics in the intended curriculum up to the high school level. It is this content of school algebra that teachers are expected to help students discover or learn in their algebra classes. In their work, researchers in the KAT project set the limits of this type of knowledge by reviewing content standards of ten different states in the US. At the Senior High School level in Ghana, the content of this type of knowledge, as at the time of this study, is included in both the Core and Elective Mathematics Syllabuses. The Kat project considers this type of knowledge as vital because in their view algebra teachers would find it difficult to influence student learning unless they comprehend the grade-level algebra content they are to teach. Since students are expected to learn their school algebra, it sounds reasonable to hypothesize that for teachers to influence students learning, they (teachers) need to understand the content of school algebra themselves.

Advanced Knowledge of Mathematics

The second type of knowledge hypothesized by the KAT project is the *Advanced Knowledge of Mathematics* (or simply "Advanced Knowledge"). This type of knowledge "includes other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini-Mundy, Senk and McCrory, 2005, p.1). To clarify the delimitation of this type of knowledge, the KAT project lists areas like calculus, linear algebra, number theory, abstract algebra, complex numbers and mathematical modeling as some of these general areas. In addition, in the conceptualization of advanced knowledge, members of the KAT project acknowledge that "knowing alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school mathematics are also characteristics of an advanced perspective of mathematics" (Ferrini-Mundy, Senk and McCrory, 2005, p.1). Thus, in simple terms, this type of knowledge includes school content that precedes algebra (such as sets of numbers, operations on numbers etc.), as well as content that proceeds algebra.

The KAT project considers possession of Advanced Knowledge as important because it could afford teachers with a deep or profound understanding of school algebra. This is a result of a number of reasons. First, with this type of knowledge teachers could make connections across topics while unpacking the complexity of any mathematics content to make that content more understandable by students. Second, in class, the KAT project hypothesizes that a teacher's task, may also involve unpacking content preceding the content of focus; and the possession of Advanced Knowledge makes this possible. Third, the KAT project identifies the three major processes that could be vital to effective teaching. These are bridging, trimming and decompressing of the content of school algebra to students. The project defines bridging as making connections across topics, trimming as removing complexity while retaining integrity and decompressing as unpacking complexity to make content more comprehensible. It is hoped that since teachers who possess Advanced Knowledge have a deep or profound understanding of school algebra (from having a good knowledge of the trajectory of the content of school mathematics), such teachers would teach effectively since they can succeed in helping their students through the processes of bridging, trimming and decompressing. In other words, according to members of the KAT project, possession of Advanced Knowledge could make one teach effectively because of the ability to fluidly engage students to make connections across topics (i.e., bridging), remove complexity while retaining integrity of the algebra they teach (i.e., trimming) and be able to unpack complexity to make content of school algebra more comprehensible (i.e., decompressing) to students.

Teaching Knowledge

The third category of knowledge in the KAT framework is the Teaching Knowledge. According to the KAT researchers, "the knowledge referred to here may fall into the category of pedagogical content knowledge or it may be pure mathematical content applied to teaching" (Ferrini-Mundy, Senk & McCrory, 2005, p.1). The KAT project members throw further light on this by saying that, this knowledge is described as

> "knowledge specific to teaching algebra that may not be taught in advanced mathematics courses. It includes such things as what makes a particular concept difficult to learn and what misconceptions lead to specific mathematical errors. It also includes mathematics needed to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to emphasize with curricular trajectories in mind and to enact other tasks of teaching" (Ferrini-Mundy, McCrory, Senk & Marcus, 2005, p.2).

From this, the point can be made that since this type of knowledge may not be taught in advanced mathematics courses, it may not necessarily be available to mathematicians. Consequently, this is the knowledge that could differentiate an engineer or a mathematician from an algebra teacher. It is as if to say that this type of knowledge is the unique type of knowledge teachers have and which they use in the teaching the subject matter of school algebra better than anybody with only good content knowledge, whether school or advanced knowledge.

The Relationship among the three hypothesized knowledge types

As already indicated, these three types of knowledge, discussed above, constitute the theoretical frame of algebra knowledge for teaching that guided this study. The KAT project conceptualizes these hypothesized knowledge types are not hierarchical in nature. In addition, they do not exist in a continuum with well-definable boundaries. Instead, their boundaries are blurry in the sense that they are interwoven in many ways. A schematic diagram of this conceptualization is presented in Figure 1 below.

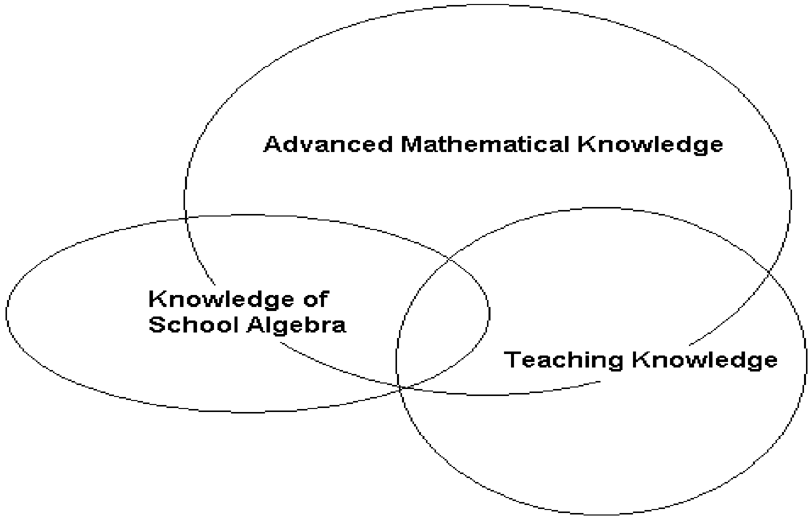

Figure 1: Conceptual Representation of the KAT project's Three Types of Knowledge

Instrumentation

Two instruments, Form 1 and Form 2 were developed by the Knowledge of Algebra for Teaching (KAT) project team in their validation study. Each of these forms comprised 20 items in all; 17 multiple choice items and 3 open-ended items. The difficulty levels of the two forms have been shown to be comparable in an earlier study (see Wilmot, 2008). In this study, therefore, only one of the two forms, Form 1, was adapted and used for data collection. The adaptations involved changing the US context in the original instrument into Ghanaian contexts. For instance, not only was the US currency changed into the Ghanaian currency, the prices of the items were also changed to reflect market values in Ghana at the time of the study. In addition, variations in names commonly used for the commodities used in the item were also changed to reflect the right contexts in Ghana. The instrument did not require the use of any identifiers like names, gender, school, identity numbers or anything that could be traced to participants.

Procedures

Participants of this study comprised prospective senior high school mathematics teachers (i.e., final year students majoring in mathematics and mathematics education) from two universities in Ghana, as well as in-service teachers in Senior High Schools in four regional capitals in Southern Ghana. The study lasted for four weeks. Within the first two weeks, visits were made to the two universities and the senior high schools within the four cities. In the Universities, meetings were held with the Heads and lecturers of the Departments in which Mathematics Education and Mathematics were housed. In the Senior High Schools, similar meetings were held with the Heads of Institutions, Head of Mathematics Departments and Mathematics teachers of both Core Mathematics and Elective Mathematics. Dates for these study site visits and meetings had earlier been fixed through prior telephone calls made to the Heads of Departments in the Universities and the Heads of the Senior High Schools. The telephone calls provided opportunities for a timeline to be agreed upon for initial visits to meet with participants of the study and the Heads of Departments and Schools.

During the initial visits and meetings, the rationale of the study was discussed and approvals were obtained to use the institutions as sites for the study. Consent was also obtained from the university students and in-service teachers to participate in the study. At each site, two copies of consent forms were signed by the Head and students or teachers as the case may be. One copy of the signed form was collected back while the other was meant for their records. This provided opportunities to know how to plan for the administrations of instruments and how many of the instruments to be printed. In addition, the meeting also provided opportunities for timeline for the data collection to be completed.

As already discussed, this study involved administering the instrument adapted from the KAT project to final year mathematics and mathematics education students in two universities, as well as mathematics teachers in eight senior high schools in Ghana. Administration of instruments in the senior high schools was conducted during the last two weeks of the school year. In each senior high school, the instrument was administered after the normal school hours so as not to disrupt classes. Also, in each senior high school, the in-service mathematics teachers who agreed to participate were brought together to complete the instruments at a sitting lasting no more than 60 minutes. From the eight senior high schools, 20 in-service mathematics teachers participated in the study.

Administration of the instruments in the universities was done in a slightly different manner. The universities were writing their end-of-semester examinations during the period of data collection. These examinations notwithstanding, the heads of the participating departments had agreed with the participating students to incorporate the administration of the instruments with the examinations. Consequently, opportunity was provided during the examination weeks, for students who agreed to participate to come together to complete the instruments on the days that they did not have any examination to write. This made it possible for participants in each department of the two universities to complete the instrument at a sitting lasting no more than 60 minutes. In the two participating universities, 189 prospective senior high school mathematics teachers in Ghana took part in the study Thus, altogether 209 participants completed the instrument used in the study.

Scoring of Content Items

Responses to the multiple choice items were scored as right (1 mark) or wrong (0 mark) while the open-ended items were scored on a four-point scale. The following is ae summary of the main features of the rubrics used for scoring the open-ended items since the exact scoring rubrics could not be presented in this publication:

Score of 4: All steps of the solution have carefully been laid. A reason for each step does not necessarily have to be given but each step follows reasonably from the one before. The solution can be shown as a model solution to any audience.

Score of 3: All steps of the solution have carefully been laid but there are minor errors.

Score of 2: There is an evidence of a chain of reasoning but some major conceptual mistake was made or there is an evidence of a chain of reasoning but the solution is not complete.

Score of 1: There is at least one correct statement.

Score of 0: Something mathematical is said but is not valuable for the question.

Score 777: Nothing mathematical is said (e.g. "no clue", "I don't Know")

Score 999: Blank

Mode of Data Analysis

The research question that guided this study was,

> "To what extent does Ghanaian pre-service and in-service secondary mathematics teachers' knowledge for teaching algebra corroborate the three categories of knowledge hypothesized in the KAT framework?"

To answer this question, data from the university students and in-service teachers were used. Altogether, 209 participants completed the instrument. In theory, the number of variables or factors needed to explain the variation in the data could have been modeled by using structural equation modeling (SEM) and factor analysis could have been incorporated in SEM to confirm these variables. However, SEM could not be used in this analysis because of the following reasons.

To use SEM, Pedhazur (1997) has argued that the subject to variable ratio must be at least 30:1. Comfrey and Lee (1992) have also suggested that to use SEM, "the adequacy of sample size might be evaluated very roughly on the following scale: 50 – very poor; 100 – poor; 200 – fair; 300 – good; 500 – very good; 1000 or more – excellent" (p. 217). On the other hand according to Gorsuch (1983) and Hatcher (1994), in Exploratory Factor Analysis, a subject to item ratio of at least 5:1 is recommended while Nunnally (1978) argues for a ratio of 10:1.

Since 209 subjects participated in this study and there were 20 items, only the Gorsuch (1983) and Hatcher (1994), as well as the Nunnally (1978) criteria were met (see also, Nunnally & Bernstein, 1994). Therefore a decision was taken that Factor Analysis, as a stand-alone test, was the best for the study.

Exploratory factor analysis was performed on the data collected. Factor analysis was chosen because it helps, among other things, to examine the number of variables, called factors, which could be used to either completely or to a large extent explain the variation in scores in the data collected. In the conceptual framework, three types of knowledge had been hypothesized. However, in this study, no prior assumption was made about the truth or otherwise of this hypothesis. In other words, no specific decision was made earlier in this study about the exact number and nature of the underlying factor structure (i.e., of the type of knowledge measured by the instrument). Consequently, Exploratory Factor Analysis was used. The idea was to allow as many factors as items on each of the instruments to be extracted so that a decision could be made, based on the factor loadings, as to the number of factors that could be retained to explain the pattern of relationship among scores in the data. It helped to answer the question of whether there is enough evidence to conclude that three factors could be distinguished, a number corresponding to the types of knowledge hypothesized in the theoretical framework. In addition, exploratory factor analysis helped to determine whether the factors that emerged could be described, using the three types of knowledge hypothesized in the framework. The extraction method used was principal component analysis and the rotation method used was *Oblimin* with Kaiser Normalization. Oblimin rotation was used because of its ability to allow the factors extracted to be correlated. In the next sub-section, results of the factor analysis are presented and discussed.

Analyses and Findings

The first step in the factor analysis was the examination of the number of factors needed to explain the variation in scores on the various items on Form 1. Table 1 below, shows how items loaded on various factors and the variance explained by all possible factor loadings when Factor Analysis was done to retain three factors. This table, Table 1, shows results of the number of possible factors that could be extracted from the data to explain the variation among the scores and their corresponding eigenvalues. The eigenvalues give an indication of the strength level of each of the extracted factors. Consequently, the eigenvalues could be used to decide on the required number of factors needed to represent the relationships in the data.

Table 1 Total Variance Explained by Each of the Factors

Factor	Initial Eigenvalues		
	Total	% of Variance	Cum. %
1	2.526	12.628	12.628
2	2.346	8.244	20.872
3	2.159	7.342	28.214
4	1.849	6.838	35.052
5	1.668	6.520	41.573
6	1.468	5.978	47.551
7	1.126	5.686	53.236
8	1.056	5.281	58.517
9	0.992	4.962	63.479
10	0.94	4.700	68.179
11	0.882	4.412	72.591
12	0.795	3.924	76.516
13	0.755	3.523	80.039
14	0.699	3.495	83.534
15	0.638	3.192	86.726
16	0.61	3.048	89.774
17	0.569	2.981	92.755
18	0.498	2.492	95.248
19	0.482	2.409	97.656
20	0.469	2.344	100.000

In Table 1, a low eigenvalue for a given factor implies that factor's contribution to the explanation of variances in the variables is small and may be ignored. Consequently, in this analysis, the Kaiser-criterion (also referred to as the K-1 rule) of retaining only the factors with eigenvalues greater than 1.0 was initially considered. Based on these initial eigenvalues (see the second column of Table 1), it is could have been concluded, according to the K-1 rule, that eight main factors could be retained. Together these eight would have explained about 58.5% of the variance. However, since the theoretical framework guiding this study hypothesizes three main knowledge types, the eight factors revealed by the Kaiser criterion was initially held tentative and the scree-test plot used for further check.

Essentially, scree-test plots are graphs of the factors (as shown in Table 1 above) on the horizontal axis against the corresponding eigenvalues on the vertical axis. On this graph, as the number of factors increases (i.e., as one moves from left to right along the horizontal axis), the eigenvalues decrease (refer to this also from Table 1). However, the change in slope of the graph resulting from these variations is usually not constant but decreases as the number of factors increases. Conventionally, the steepness of the slope of various sections of the graph is examined and the x-coordinate of the point beyond which the variation in slope begins to be somewhat uniform (i.e., the elbow of the graph) is chosen as the needed number of factors. The graph below, Figure 2, shows the scree-test graph obtained from the factor analysis.

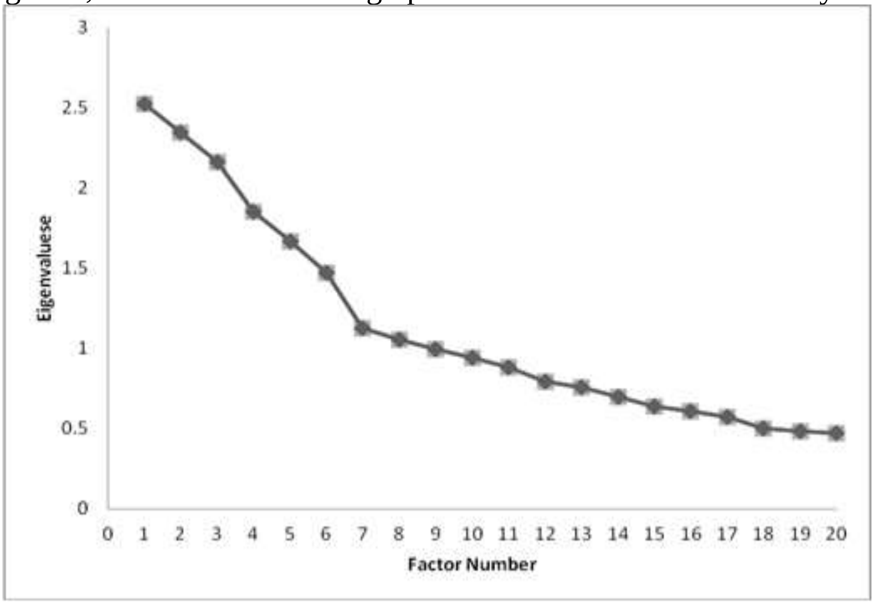

Figure 2: Scree plot of the factor loadings

From Figure 2, it will be observed that a moving from left to right on the graph, the variation in the steepness of the graph reduces relatively more beyond factor 7. In other words, the elbow of the graph can be seen to exist at factor number 7. Hence, it was concluded from the scree-test that seven factors can be said to be retained for further analysis. Thus, whereas the factor analysis extracted 20 factors for examination (because of the 20 content items on the research instrument), it was inferred from the scree plot that the number of factors needed to explain the variation in scores in the data was seven.

It can be seen from Table 1 that these seven factors together explain about 53.2% percent of the variation in scores.

Consequently, the factor loadings were examined using seven factors. To interpret the factor loadings, loadings of absolute value above 0.30 were considered strong enough to be indicative of the nature of the factor. Also, since cross loading (i.e., loading of 0.30 or above on more than one factor) is indicative that an item cannot be uniquely assigned to any of the factors, such items were removed and not used to determine the nature of that factor (see Guadagnoli & Velicer, 1988).

Table 2: Item Loadings on the Seven Retained Factors

	Factors						
Item	1	2	3	4	5	6	7
1**	0.62	-0.01	-0.14	0.33	-0.18	0.07	0.22
2	0.28	-0.04	-0.10	0.05	-0.12	0.10	0.71
3	0.17	0.16	0.24	-0.06	0.04	0.17	-0.72
4	0.09	0.28	0.15	0.60	0.07	-0.29	0.17
5**	0.39	0.42	-0.10	-0.25	-0.23	-0.07	-0.08
6**	0.32	-0.10	0.05	0.05	-0.49	0.54	0.17
7	0.10	0.10	0.76	-0.12	-0.06	0.10	-0.01
8	0.10	0.08	0.19	0.07	-0.82	0.06	0.06
9	-0.13	-0.17	0.14	0.60	-0.25	0.01	-0.27
10	0.05	0.61	0.04	-0.08	-0.05	0.01	0.07
11	0.25	0.60	0.17	0.08	0.09	0.19	0.28
12	-0.03	-0.07	0.19	0.70	0.13	0.16	0.19
13	0.03	0.02	0.65	0.26	-0.07	-0.13	0.10
14	-0.11	0.12	0.00	0.00	0.02	0.79	0.01
15**	0.38	-0.01	-0.17	0.05	-0.01	0.02	-0.61
16	0.72	0.10	0.19	-0.06	0.00	0.01	0.11
17**	0.39	0.11	0.38	0.13	0.46	0.32	-0.06
18**	-0.25	0.05	0.02	-0.03	0.65	-0.04	-0.35
19	0.70	0.02	0.06	0.07	-0.06	0.07	0.04
20**	0.18	-0.13	-0.12	0.58	0.07	0.35	0.07

** Items with cross loadings

Next it was necessary to examine the manner of loadings of the items on the seven factors. The rationale was to determine whether such factor loadings could help name or describe each of the seven factors. To simplify the discussion with these seven factors, the strongly loaded items on each of the factors have been extracted and presented with the way the way categorized by the KAT project members in Table 3 below. On this Table (i.e., Table 3), items that had cross loadings (i.e., those that loaded strongly on more than one factor) have been excluded.

Table 3: Item loadings and categorization on the seven factors

Factors	Number of Items	Item ID and Categorization (in parenthesis)		
1	2	16 (Adv. Kn)	19 (Sch. Kn)	
2	3	5 (Tchg. Kn.)	10 (Tchg. Kn)	11 (Tchg. Kn)
3	2	7 (Tchg Kn)	13 (Adv. Kn)	
4	3	4 (Adv Kn)	9 (Adv. Kn)	12 (Adv. Kn)
5	1	8 (Tchg Kn)		
6	2	3 (Sch. Kn)	14 (Sch. Kn.)	
7	1	2 (Tchg. Kn)		

As shown in Table 3, two factors, Factors 5, and 7 had only one item each loading strongly on them. In addition, three factors, Factor 1, 3 and 6 had only two items each strongly loading on each of them respectively. The remaining two factors, Factor 5 and 7 had only one item loading strongly on them. Again, one or two item loadings were considered too small to be indicative of the nature of these factors. As a result, five of the seven retained factors (i.e., Factors 1, 3, 5, 6, and 7) were considered too unstable to be correctly interpreted in this study. The only factors on which the minimum of three items loaded were Factors 2 and 4. On Factor 2 all the three items that loaded strongly (i.e., items 5, 10, and 11) were previously categorized by the KAT project as Teaching Knowledge items while the other three that loaded on Factor 4 (i.e., items 4, 9 and 12) were all Advanced Knowledge items. Thus, only two of the seven retained factors could be labeled, if necessary in terms of two of the types of knowledge hypothesized in the KAT framework, which guided this study. These are the *Teaching Knowledge* and the *Advanced Knowledge* sub-categories of knowledge.

Discussions and Conclusions

In this section, the findings of the study are discussed alongside their corresponding conclusion. Two of these that are quite general are presented first and the major conclusion, an expansion in the KAT framework, is separated and presented under separated subheading. This is because, it is the study's most significant contribution to the literature on conceptualization of teacher knowledge. It is hoped that presenting this would make it stand out more conspicuously.

Generally, the KAT project's conceptualization of teacher knowledge is important on two counts. It attempts to focus on the idea of teacher knowledge especially at the high school level in domain specific terms. In addition, the conceptualization lends itself to the idea of being able to develop instruments to assess the hypothesized knowledge types.

However, the results of this study only partially but not completely corroborate the KAT framework that guided the present study. In spite of this, the findings point to the following issues that need to be considered when discussing teacher knowledge in domain specific terms.

The fact that Factors 2 and 4 had only *Teaching Knowledge* and *Advanced Knowledge* items loading respectively on them point to the fact that these two Factors could be named *Teaching Knowledge* and *Advanced Knowledge* respectively. In addition, it is worth noting that on Factor 6 the only two items that loaded were all *School Knowledge* items. Factor 6 was however not labelled as the *School Knowledge factor* because of the suggestion by Costello and Osborne (2005) that factors with fewer than three items are considered to be unstable and should not be labeled. Thus, this study partially corroborates the hypothesized knowledge types in the KAT framework. This notwithstanding, data from this study can be said to somehow point to the possible existence of these three hypothesized knowledge types in the KAT framework. This study confirms the existence of two of the KAT project's hypothesized knowledge types but not all three. Further studies are therefore needed to fully confirm the KAT framework.

Also, in the face of the fact that 6 out of the 20 items on the KAT instrument adapted in this study had cross loadings, leaving only 14 items to load on the seven extracted factors, it is obvious that there are not enough items on the instrument. That explains why some items had only one items loading on them. Gorsuch (1983) prefers six variables per factor and suggests a minimum of four variables per factor only in situations where the factors have been exceptionally defined in previous research. Using this criterion means that running a factor analysis using seven factors would need an instrument of at least 42 items, even when there are no multiple loadings. The implication is that further studies in this direction needs to develop more items to add to the KAT items for use. When this is done, it is hoped that the single and two item loadings on some of the factors could be eliminated. Consequently, the factor loading may be able to reveal the true nature of all the factors and possibly corroborate the expanded framework suggested in the proceeding paragraphs.

The expanded KAT framework

The KAT project team members have earlier conjectured that their three hypothesized knowledge types did not exist in a continuum and that they are interwoven in many ways. Consequently, according to them, the intersections are blurry. This is what led to the schematic diagram of three concentric circles (refer to *Figure 1*). The blurry nature of their intersection points to the fact that their intersections are unclear and possibly not significant to be focused on or not necessary to be defined. However, the findings of this study point to the possibility that the boundaries of the three hypothesized knowledge may not blurry as initially conjectured in the KAT framework. This is because results of the current study have revealed that an Advanced Knowledge item and a School Knowledge item have loaded together on one of the extracted factors (i.e., Factor 1) while a Teaching Knowledge item and an Advanced Knowledge item have also loaded on another factor (i.e., Factor 3).

This warrants the need for an expansion to be made in the KAT framework in a way that permits the intersections of the original three KAT project's hypothesis not to be regarded as blurry but as well defined complex interactions of the three types of knowledge by teachers. This is an important contribution to the discussion of teacher knowledge in domain specific terms that has implications for future research in this direction. It is a view consistent with what Putnam (1987) calls curriculum scripts made up of interrelated set of organized actions. The argument here in this recommended expanded framework is that the notion of how teachers transform their knowledge into pedagogical representations could be seen in a multifaceted combination of the knowledge they possess. A schematic diagram of this expanded conceptualization suggested from the findings of the study is as shown in Figure 3.

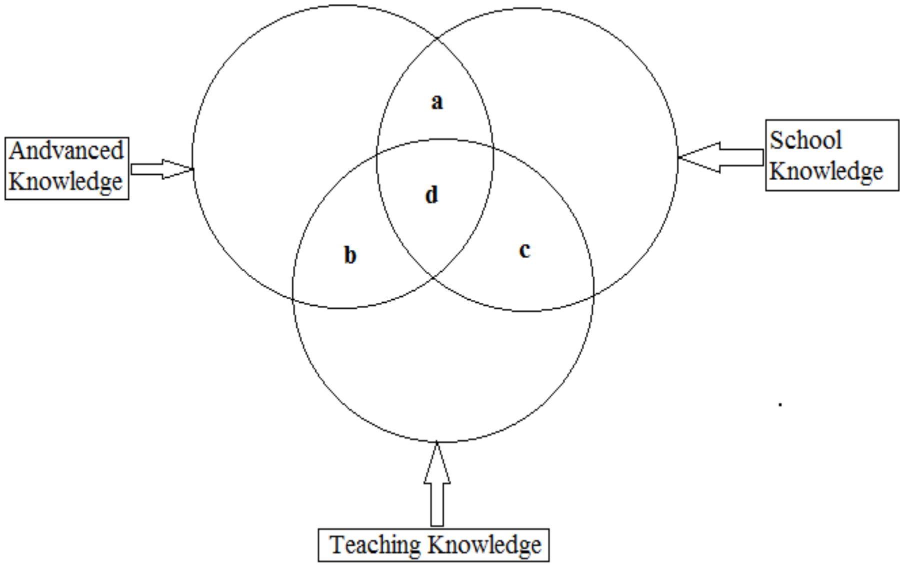

Figure 3: Schematic suggested expanded framework from the findings of the study

In this suggested expansion in the KAT framework, due to the fact that the findings of this study could not directly reveal the nature of the the intersections of the KAT project's hypothesized knowledge types or caused them to be properly defined, they have been labeled as ***a, b, c*** and ***d.*** They can be hypothesized based on their position relative to the three original knowledge types as shown in *Figure 4.*

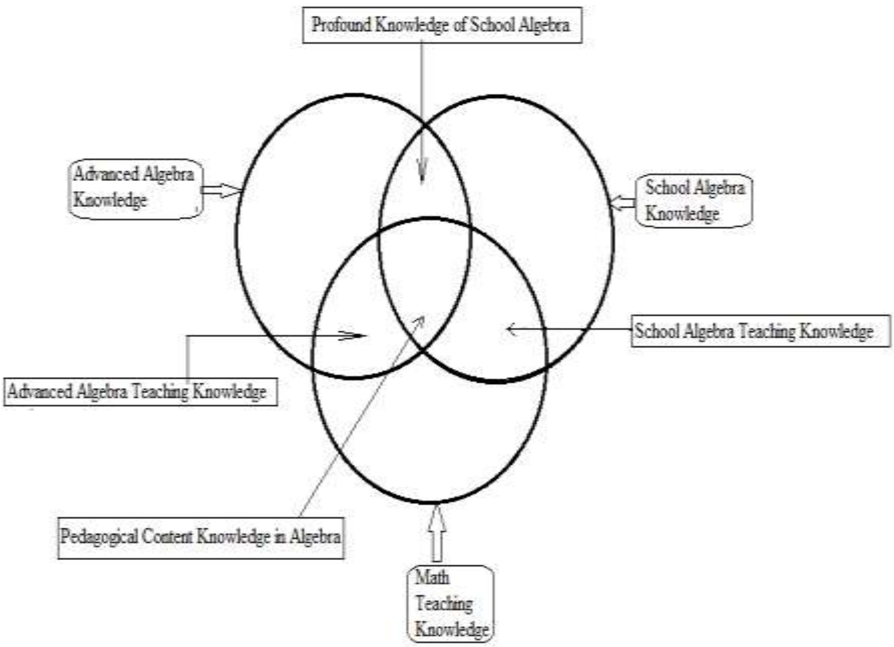

Figure 4: Expanded framework for reconceptualization of domain specific teacher knowledge

The main categories of knowledge, School Algebra Knowledge, Advanced Algebra Knowledge and the Algebra Teaching Knowledge are the same categories of knowledge hypothesized respectively in the KAT framework as, Knowledge of School Algebra/School Knowledge, Advanced Knowledge of Mathematics and Teaching Knowledge except for two main considerations. First, in the present conceptualization they are renamed to make them domain specific (i.e., specific to algebra on which the instruments were developed). Second, some of the exceptions recommended for re-categorization of the KAT framework is what have been used to define the intersected categories to make the intersections distinct in the sub-sections. The various categorized knowledge types formed as a result of the intersections in the schematic diagram, Figure 4, are discussed in the subsections that follow.

Profound Knowledge of School Algebra

As the name implies teacher who possess this type of knowledge can be said to have a deep understanding of school algebra. This may include possession of alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school algebra. Content that precedes school algebra, as well as those that proceeds it are part of this category of knowledge.

School Algebra Teaching Knowledge

A teacher who possesses this type of knowledge have a good knowledge of the trajectory of school algebra. Having this type of knowledge is crucial because it allows teachers to teach algebra in a fluid manner to enhance understanding of diverse groups of learners. It is this is type of knowledge that enables teachers to engage in bridging (make connections across topics in school algebra), trimming (removing complexity while maintaining integrity) and decomposition (unpacking complexity to make content more comprehensible) while teaching school algebra.

Advanced Algebra Teaching knowledge

Teachers who possess this type of knowledge, do not only have a good understanding of advanced algebra, but also how to teach it effectively. Similar to the conceptualization of the School Algebra Teaching Knowledge category, the Advanced Algebra Teaching Knowledge is this type of knowledge that enables teachers to engage in bridging (make connections across topics in advanced algebra courses). Teachers who possess this type of knowledge are also able to engage in trimming and decomposition when it becomes necessary for them to teach advanced algebra courses.

Pedagogical Content Knowledge in Algebra

This is the type of knowledge initially conceptualized by Shulman (1986b) as involving a complex combination of some form of content and pedagogical knowledge except that unlike Shulman's conceptualization is a generalized one, the conceptualization in this expanded framework is a domain specific type of knowledge (i.e., specifically connected with algebra).

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