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THE EFFECTS OF USING MANIPULATIVES IN TEACHING AND LEARNING OF ALGEBRAIC EXPRESSION ON SENIOR HIGH SCHOOL (SHS) ONE STUDENTS' ACHIEVEMENTS IN WA MUNICIPALITY

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Abstract

The study set out to investigate the effects of using manipulatives in teaching and learning of algebraic expressions on Senior High School one students' achievement in the Wa Municipality. The purpose was to find out what type of manipulative is being used to teach algebraic expressions and its effects on the achievements of students in Senior High Schools. In all, 10 mathematics teachers were randomly sampled from all the senior high schools in the Wa Municipality while eighty students were also selected from the same schools for the study. The research design used for the study was a survey and the instruments used for collecting the data on the study were achievement test and questionnaire. An ANOVA test conducted revealed that there was a significant difference in achievement between students taught algebraic expressions using manipulatives and those taught without using manipulatives. The study also found that students were very active and in high spirit to learn mathematics when manipulatives were used. The study further found that students taught algebraic expressions with the use of manipulatives achieved higher than those taught without the use of manipulative. The study concludes that teachers should always use manipulatives to teach concepts in mathematics since it is pupils friendly, activity oriented, arouses students' interest and facilitates higher understanding that results in higher achievement.

Keywords: algebraic expression; effects; investigation; manipulatives; students' achievement.

Introduction

Until fairly recently, algebra was considered an exclusively letter symbolic domain. Much of the research that was conducted during the period preceding the 1990s focused specifically on the transition required by students as they moved from arithmetic to algebra at about age thirteen or fourteen (Kieran, 2001). Kieran notes that this large body of research which continues to grow has investigated the learning of concepts that under-pin students' successes in algebra. These concepts include unknowns and variables, expressions and equations and the expansion of the meaning given to the equal and minus signs. Some of the earliest research focused on students' ability to discriminate among the different ways that letters are used in algebra (Nathan & Koedinger, 2000). However, changing curricular emphases and using technological tools have expanded students' views of algebraic letters.

Spread sheet activity has been found to encourage simultaneous multi-valued and single-valued interpretations of the letter (Ainley, 2002). The shift in the content of algebra from equation centered to function-centered content has broadened students' views of algebraic letters, but has also introduced additional difficulties (Chazana &Yerushalmy, 2003). Healy and Hoyles (2005) observe that the use of patterning activities to develop meaning for algebraic expressions suggests that hard work is needed by students in order for them to express the observed numerical and geometric patterns in a letter-symbolic form. Many of the difficulties that students face in learning algebra may have their source in the poor understanding of two important concepts: the variable and the algebraic expressions.

Over the years, the role of the teacher has changed from being the transmitter of knowledge to being the facilitator of learners' discovery of knowledge. This means that learners' roles have equally changed from being spectators in the game of learning to being active participants. It appears that conceptual understanding of algebra is essential, and methods that enhance this understanding should be included in the mathematics curriculum. The poor understanding of algebra has multiple effects on students' performance in mathematics.

To overcome the challenges in learning mathematics, a variety of teaching and learning strategies have been advocated for use in mathematics classrooms, moving away from the teacher-centered approach to more student-centered ones, and the use of manipulative materials is one of such potential strategies. Also, there is a substantial evidence of research work done by other researchers on the various effects on the use of manipulatives on students. For example, Cramer (2002) indicates that girls favour and achieve higher in cooperative learning than in competitive learning.

The majority of recent research supports the importance of using concrete materials in developing mathematical concepts (Dienes & Golding, 1971; Reys, 1971; Suydam & Dessart, 1976). Suydam and Higgins (1976) in their study of the activity-based mathematics learning in grade k-8 determine that mathematics achievement increased when manipulatives were used. Suydam (1984a) suggests that manipulatives enhance mathematics achievement across a variety of topics, grade levels, achievement and ability levels.

Statement of the Problem

Evidence of poor performance in mathematics by Secondary School students' points to the fact that the most desired technological, scientific and business application of mathematics cannot be sustained. This makes it paramount to seek for a strategy for teaching algebra that aims at improving students' understanding and performance in it. Evidence abound (Srinivasa, 1978; Ogunkunle, 2000) that lack of mathematics teaching aids and Mathematics teachers' non-use of manipulatives in teaching mathematics is one of the major factors that contribute to poor achievement in mathematics by secondary school students.

At the basic level of education as pointed by Mereku (2001), the Ghanaian mathematics teacher is regarded as a demonstrator of process and transmitter of information and teaches largely through lecturing and teacher-centered approaches. This prevents the students from experiencing the learning of mathematics using manipulative materials. No wonder therefore, that students' performance in mathematics in Ghana remains among the lowest in Africa and the world (Kraft, 1994; TIMSS, 2007). In Ghana, most students learn without adequate teaching and learning materials (Nabie, 2009). The use of teaching and learning materials in teaching enables the child to form a concept. Nabie (2009), cited in Bolton (2010), describes concept formation as a process in which a person recognizes similarities and abstracts the resemblances away from the other properties that are not relevant to the concept.

Many students have struggled with understanding mathematical concepts and become frustrated in the classroom (Rust, 2008). Most students have low-test scores and have difficulty in completing homework (Rust, 2008). It becomes apparent for the need to research into the use of manipulatives in the Mathematics classroom. Therefore, a study to find out the effects of using mathematics teaching aids in teaching on the achievement of Senior High School (SHS) (1) mathematics students is of great relevance. This may go a long way to improve the teaching and learning of algebraic expressions. It will be more relevant to investigate the effect of using manipulative materials in the teaching and learning of algebraic expressions in Wa Municipality in Upper West of Ghana.

Research Questions

The following research questions were formulated:

1. To what extent are manipulative materials used during algebraic expression lessons in the Wa Municipality?
2. What is the level of participation of students during algebraic expressions lessons using manipulative materials in Wa Municipality?

Hypothesis

H_0: There is no significant difference in achievement between students taught algebraic expressions using manipulative materials and those taught without using manipulatives.

History of Manipulatives

Since ancient times, people of different civilizations have used physical objects to help them solve everyday mathematics problems. The ancient civilizations of Southwest Asia used counting boards, which were wooden or clay trays covered in a thin layer of sand. The counting board users would draw symbols in the sand to tally inventory or whatever else they may need to count. The ancient Romans created the first abacus based on counting board. The abacus was made of beans or stones which moved in grooves in sand or on tables of wood, stone, or met. al. The Chinese abacus, which came into use centuries later, may have been an adaptation of the Roman abacus. The Mayans and the Aztecs both had counting devices that were made of corn kernels strung on string or wires that were stretched across a wooden frame.

The late 1800s saw the invention of the first true manipulative-maneuverable objects that appeal to several different senses and are specifically designed for teaching mathematical concepts. In 1837, German educator Friedrich Froebel introduced the world's first kindergarten. He designed the educational play materials known as

Froebel Gifts, or Frobelgaben, which included geometric building blocks and pattern activity blocks (Friedrich Froebel, 2009). Then in the early 1900s, Italian educator Maria Montessori continued with the idea that manipulatives are important to education. She designed several materials to help elementary students learn the basic ideas of maths. Since the 1900s, manipulatives have come to be considered essential in teaching mathematics at the elementary school level. In fact, the National Council of Teachers of Mathematics (NCTM, 2000) has recommended the use of manipulatives in teaching mathematical concepts at all grade levels (Matthew *et al.*, 2010). The use of manipulatives to teach algebraic expressions can greatly increase students' understanding and learning progression of concept (Rapp, 2009). According to Hartson (2006), manipulatives are objects that can be touched and moved by students to introduce or reinforce a mathematical concept.

During the 1960s and 1970s researchers compared, in a number of educational settings, outcomes of Mathematics instructions with concrete materials or pictorial materials to outcomes of instructions without such materials. The results were often mixed. Findings in some comparisons favored the group using the materials, whereas in other comparisons the control group achieved comparable or better results. Some early reviewers of research on manipulatives simply summarized findings and let readers draw their own conclusions about the effectiveness of the materials. Others concluded that manipulative materials were beneficial for young children but were unnecessary for older children (Fennema, 1972; Friedman, 1978; Johnson, 1971; Kieren, 1969; Scott & Neufeld, 1974). Kieren, (1971) claimed that students learn Mathematics well in laboratory settings where manipulative materials are common, but that other methods of instruction work equally well.

The Level of Participation by Students During Lessons on Algebraic Expressions Using Manipulative.

Children are naturally curious, playful and full of energy. Sousa (1995) reports that children do not often enjoy sitting for extended periods of time, listening to their Mathematics teacher lecture. Beyond the lack of enjoyment, most students in a sit-and listen mathematics lesson walk away with low degree of understanding and retention. Sousa further indicates that utilizing manipulative materials allows children to break away from the traditional classroom setting and instructional style. Using manipulatives can be exciting and motivating to students, naturally leading toward a greater interest in the intended use of manipulatives and the learning activity.

Moyer (2001) studied 10 teachers, focusing on how and why they used manipulative materials in their classrooms. While the teachers who participated in the study claimed that the manipulative materials were fun but not necessary to teaching and learning mathematical concepts, there was an overwhelming positive behavior exhibited by students when using the manipulative materials. Moyer finds that in lessons where manipulatives were used, students appeared to be interested, active and involved.

In addition to the ability of manipulatives to aid directly in the cognitive process, manipulatives have the additional advantage of engaging students and increasing both interest in and enjoyment of mathematics. Students who are presented with the opportunity to use manipulatives report that they are more interested in Mathematics. Long-term interest in mathematics translates to increased mathematical ability (Sutton & Krueger, 2002). Young (2004) supports the NCTM contention that physical materials have a positive effect on students understanding and involvement during Mathematics lessons.

Achievement Rates of Students Taught Algebraic Expressions Through the Use of Manipulatives.

Bloom, Hill and Lipsey (2008) declare that our goal in teaching Mathematics is to have students understand and apply Mathematics to the everyday world. Students understanding can only come when they have been actively involved in their own learning. Students must do mathematics. They need to take charge of their own learning and teachers must show them how, and provide them with the opportunities to do so. Ozel (2009) supports the idea that manipulatives can help students and teachers to bridge the gap that divides how Mathematics is taught and how Mathematics is learned. According to Konold (2004), manipulatives used in the classroom can help students at all grade levels to understand processes, communicate their mathematical thinking and extend their mathematical ideas to higher cognitive levels during algebraic expression lessons.

The use of manipulatives helps students hone their mathematical thinking skills during algebraic expression lessons. According to Steen, Brooks and Lyon (2006), "Manipulatives can be important tools in helping students to think and reason in more meaningful ways in algebraic expression lessons. By giving students concrete ways to compare and operate on quantities such as manipulative materials, pattern blocks, tiles and cubes, can contribute to the development of well-grounded, interconnected understandings of mathematical ideas"

Manipulatives are especially useful for teaching low achievers, students with learning disabilities and learners of English language (Ruzic & O'Connell, 2001). To gain a deep understanding of mathematical ideas such as algebraic expressions, students need to be able to integrate and connect a variety of concepts in many different ways. Clements (1999) calls this type of deep understanding "integrated-concrete" knowledge. The effective use of manipulatives can help students connect ideas and integrate knowledge so that they gain a deep understanding of mathematical concepts. With long-term use of manipulatives in Mathematics, educators have found that students make gains in several areas such as relating real-world situations to mathematical symbolism (Sebesta & Martin, 2004). Studies have shown that students using manipulatives in specific mathematical areas such as algebraic expressions are more likely to achieve success than students who do not have the opportunity to work with manipulatives. (Sebesta & Martin (2004); Chappell & Strutchens, 2001).

According to Chappell and Strutchens (2001), students who used manipulatives in their mathematics classes had higher algebraic abilities than those who did not use manipulatives. Heuser (2000) indicates that using manipulatives helps improve the environment in mathematics classroom during algebraic expression lessons. Keeping students engaged and motivated to learn mathematical concepts is another challenge for teachers. At the basic level of education as pointed by Mereku (2001), the Ghanaian mathematics teacher is regarded as a demonstrator of process and transmitter of information and taught largely through lecturing and teacher-centered approaches.

In a comprehensive review of activity based-learning in Mathematics, Agashi (2003) concludes that using manipulative materials always produces greater achievement gains than not using them. In a similar study that compares the effects of using manipulative materials in teaching with that of abstract teaching in a mathematics class, Kurumeh and Achor (2008) observe that the long-term use of manipulative materials by teachers improves students' achievement and attitude. In fact, research shows that using manipulatives can contribute to the development of well-grounded and interconnected understandings of mathematical ideas. Students can more easily remember what they did and explain what they were thinking when they used manipulatives to solve a problem (Moch, 2001).

Methodology

The study is a quasi-experimental research design. Quasi-experiments are in many respects like experiments. They seek to evaluate the impact of some factors such as a particular intervention of set of factors on participants, and they aim to test hypotheses based on prior research. Quasi-experiments are founded on a positivistic paradigm which espouses the belief that it is possible to assess cause and effect, to predict outcomes and to control the effect of random, confounding or intervening variables. (Campbell and Stanley, 1963).

The population for the study was students and Mathematics teachers in all Senior High Schools in the Wa Municipality. It consisted of 15 Senior High Schools. Each SHS in the municipality had an average of ten mathematics teachers and 1500 students. The 15 SHSs in the Municipality had a total of one hundred and fifty (150) Mathematics teachers, with 22500 students' altogether. A total sample of ten (10) teachers was randomly selected from the ten (10) SHS in the Municipality which included both private and government schools. The ten schools were purposively selected based on the fact that at least each has a working population of ten (10) Mathematics teachers. For each institution, one Mathematics teacher was randomly selected or picked to answer the questionnaire.

Eighty (80) students were randomly selected from the ten selected schools. Out of the eighty (80) students, forty (40) were put into two groups of twenty (20), and treated as experimental groups while the remaining 40 students were put into two groups of twenty (20). These were also treated as control groups. The experimental groups were taught by two of the ten selected teachers using manipulatives (algebraic tiles) for a period of two weeks while the two control groups were also taught by two different teachers from the selected teachers without using manipulatives (algebraic tiles) for the same period.

In selecting the sample for the purposes of data collection, two techniques; purposive and simple random sampling techniques were used. The purposive sampling technique was used to select schools in Wa Municipality. Purposive sampling is a non-probability sampling technique where samples are chosen by intentionally seeking individuals or situations likely to provide greater understanding of a chosen concept of research (Sarantakos, 2005). Simple random sampling, however, was used to select 10 mathematics teachers and 80 students from the ten SHSs in the Municipality. Simple random sampling is a probability sampling technique where all individual participants are given equal chance of being included in the sample. As part of the methods of data collection, the instruments that were employed in the field to gather the data were mainly two. These instruments were the survey questionnaire and achievement test. These instruments were used as complements to each other with the aim of compensating for scantiness and gaps. This depicts the idea of triangulation of data collection instruments as means of enhancing the validity and reliability of data that were gathered for the study. In effect, the survey questionnaire was the instrument administered to the 10 selected mathematics teachers and the achievement test was also written by the selected students in the municipality. An achievement test was conducted during the pre-test and post-test stages of the research to determine any significant difference in achievement of students taught algebraic expressions with the use of manipulatives, and those taught without the use of manipulatives. The pre-test was conducted to find out the entry behavior of the students during lessons on algebraic expressions. The second test which was the post-test was also conducted after some lessons using manipulatives on algebraic expressions were taught.

In both the pre-test and post-test, the researcher pegged the average score at 60%. Therefore, those who scored below 60% were considered as below average and those who scored above 60% were considered to have scored above average. To establish reliability and validity, selected questions were administered to a group of thirty (30) students on two different occasions. Ten questions were given to the selected groups of students to answer. Data was collected through interviewing, test and questionnaire administration.

Validity and Reliability

In order to achieve validity and reliability, various types of triangulations were employed in the study. According to Sarantakos (2005), triangulation refers to the practice of employing several tools within the same research design. Triangulation serves several purposes in a qualitative research. It is useful for validating procedures, results and findings of the study. The triangulation strategy enables researchers to address all possible dimensions of a phenomenon, collect sufficient data for advancing knowledge and address the limitations associated with using single technique for data collection. Contextually, the validity of this study was ensured through the use of methodological, data and respondent triangulations. The adoption of these validity and reliability strategies allowed for data to be reviewed and efficiently authenticated for presentation.

Data Presentation, Analysis and Discussion

To what extent are manipulatives used during algebraic expression lessons in SHSs in the Wa Municipality?

For the researcher to find out the extent to which manipulatives are used in teaching algebraic expressions in SHSs in the Wa Municipal, a questionnaire on how often manipulatives are used by teachers was administered to the selected teachers and the responses are displayed on Table 2.

Table 2: The Extent to which Teachers in SHSs in Wa Municipality use Manipulatives in Teaching Algebraic Expressions.

Response	Number of teachers	Per centage (%)
Through-out the lesson	6	60
In the middle of the	1	10
lesson	2	20
At the beginning of the	1	10
lesson.		
At the end of the lesson		

From Table 2, it is seen that 6 teachers representing 60% of the selected teachers used manipulatives throughout algebraic expression lessons, while 40% of the selected teachers used manipulatives at the various stages of the lessons. This shows that more teachers in SHSs in the Wa Municipality use manipulatives always, which is consistent with findings of Kurumeh and Achor (2008) who observed that the long-term use of manipulative materials by teachers improved students' achievement and attitude.

What is the level of participation by SHS students in the Wa Municipality during algebraic expression lessons using manipulatives?

To assess the level of participation by SHS students in algebraic expression lessons in the Wa Municipality, the researcher gathered the responses to the questionnaire and the results are displayed on Table 3.

Table 3: Level of Participation of Students in Class During Algebraic Expressions Lesson.

Response	Number of respondents	Per centage (%)
Active	5	50
Inactive	1	10
Passive	1	10
Curious	3	30

From Table 3, 80% of the respondents indicated that the students were active and curious to learn new concepts in class when manipulatives were used in the teaching process while 20% said the students were inactive and passive. As evident in the pictorial representation of students' level of participation during algebraic lessons taught with manipulatives, more students were active and anxious to learn new ideas. This is consistent with Martins and Schwartz, (2005) who support NCTM contention that physical materials have a positive effect on students' involvement during Mathematics lessons.

Hypothesis Testing

H_0: There is no significant difference in achievement between students taught algebraic expressions using manipulatives and those taught without using manipulatives.

To test the null hypothesis, the means and standard deviations of achievement test scores were found as indicated in Table 4.

Table 4: Descriptive Statistics of Pre-test and Post-Test Scores for Control and Experimental Groups.

Test	Group	Mean	Std. deviation
Pre-test	Control	16.551	3.120
	Experimental	17.870	5.673
Post-test	Control	16.560	3.012
	Experimental	22.610	4.3356

From Table 4, it is clear that there are differences in the mean and the standard deviation (SD) of students' scores in both the pre-test and the post-test with mean (16.551) and standard deviation (3.120) as against 16.560 (mean) with (SD) 3.012 respectively for the control group, while that of the experimental group are 17.870 with a standard deviation of 5.673 as against 22.610 with SD of 4.3356 respectively.

To determine if these differences are significant, an Analysis of Variance (ANOVA) was conducted. The analysis sought to verify whether significant gains were made both within and between groups. Table 5 illustrates within groups ANOVA.

Table 5: ANOVA within Group Summary of Marks on Pre-Test and Post-Test Scores of Control and Experimental Groups.

Group	Test	Sum of squares	df	Mean square	F	P-value
Control	Pre-test-Post test	1654.9	1	50.150	1.199	0.723
Experimental	Pre-test-Post test	4681.9	1	74.316	6.863	0.002

From Table 5, it can be observed that the within group p-value for the control group was 0.723>0.05 which showed that the difference within this group was not significant, while that of the experimental group was 0.002<0.05 indicating a significant difference. This implies that students in the experimental group gained more than those in the control group. To determine any differences between group achievements, ANOVA was conducted and the results are presented in Table 6.

Table 6: ANOVA Between Groups Summary of Marks on Pre-Test and Post-Test Scores of Control and Experimental Groups.

Test	Groups	Sum of squares	Df	Mean square	F	P-value
Pre-test	Control-Experimental	89.08	1	89.08	1.100	0.789
Post-test	Control-Experimental	344.202	1	344.202	6.873	0.000

From Table 6, it can be seen that by groups, the p-value for the pre-test was 0.789 > 0.05, indicating no significant difference at the entry level; an indication that all students in both groups entered at almost the same level of knowledge. However, the p-value of the post-test of 0.000 < 0.05 indicates a significant difference in mean value. Thus, we reject the null hypothesis that: *There is no significant difference in achievement between students taught algebraic expressions using manipulatives and those taught without using manipulatives* and accept the alternative hypothesis. Hence, we conclude that there is a significant difference in achievement between students taught algebraic expressions using manipulatives and those taught without using manipulatives. This is consistent with Agashi (2003) who concludes that using manipulative materials always produces greater achievement gains than not using them

Summary of Findings

The analysis of the results of the study through the research questions indicated among other things the following:

1. The students' participatory level was very high during algebraic expression lessons taught with manipulatives.
2. Manipulative use increases students' level of understanding of operations on algebraic expressions.
3. The use of tiles in teaching algebraic expressions changed the performance of the students for better.
4. Most teachers felt because they had strong background knowledge in algebraic expressions, they could pass on knowledge to students without the use of manipulatives. This is in contrast with the studies of Ball and Bass (2000a) and Cohen (2004).

Conclusion

The findings of the research gave empirical evidence to the effects of manipulatives used in the teaching and learning of algebraic expressions. The analysis of the data did indicate a significant difference in achievement levels of student taught with the use of manipulatives and those taught without the use of manipulatives. It also suggested appropriate methods that may be employed to help develop positive interest in the teaching and learning of algebraic expressions.

The study exhibited clearly and in simple terms the approach to the teaching and learning of algebraic expressions and Mathematics in general. Pupils are always in high spirit to learn effectively when they interact with teaching and learning materials (TLMs). When teaching and learning materials (TLMs) are used, it helps to encourage and sustain pupil's interest.

Based on the findings of the study, the following conclusions were drawn. Teachers' inability to prepare and use adequate and appropriate teaching and learning materials (manipulatives) to teach algebraic expression, was a hindrance to students' performance.

- Preparation of technical terminologies in operations of algebraic expressions confuses the students. Care must therefore be taken by teachers of algebra to give enough time to students to discuss terminologies of subject-matter (especially algebraic expressions)
- Teachers' inability to encourage and entice students to develop affection for Mathematics and for that matter, algebraic expressions, was a major factor.
- The difficulty in measuring performance of students especially when they are made aware of being tested is the tendency for respondents to pretend or distort the information they provide. Hence, it is not likely for students who lack confidence in

Mathematics to respond favourable to questions by the teacher in class.

- The use of algebraic tiles could only work effectively where there was a coordinated effort from the students and their classroom teacher. Therefore, the attention of theorists, researchers and teachers is drawn to the fact that the use of algebraic tiles alone cannot improve mastery of subject-matter. Attention is also drawn to the fact that students cannot master subject-matter unless they make the effort to do so.

Recommendations

The study was done in the Wa Municipality and the following recommendations were made to help improve the teaching and learning of mathematics in schools using manipulative materials.

Curriculum developers for Senior High Schools are encouraged to stress the need for the use of manipulative materials in presenting lessons to students. They should also present in the curriculum; a suggested list of concrete materials that may be used to teach Mathematics lessons. It is recommended that further research be conducted using more students across the country to investigate whether the use of concrete materials will improve the teaching and learning of Mathematics in our schools.

The Government through the Ministry of Education should provide a motivation package in the form of allowances to be given to teachers so that they can improvise some of the manipulative materials, which are needed but unavailable in the schools. The researcher also recommends that the use of concrete materials in the teaching and learning of Mathematics should be vigorously continued to realize the maximum benefit because they enhance students' understanding, involvement, participation and interest in the learning of Mathematics.

Research on how algebraic expressions generally enables students to use algebra is mainly in small-scale teaching interventions, and the problems of large-scale implementation are not so well reported. We do not know the longer-term effects of different teaching approaches on early algebra on students' later use of algebraic notation and thinking.

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