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JUNIOR HIGH SCHOOL TEACHERS' USE OF PEDAGOGICAL CONTENT KNOWLEDGE IN TEACHING AND LEARNING MATHEMATICS IN AKATSI DISTRICT OF GHANA

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Abstract
The purpose of this study is to explore how teachers use their pedagogical content knowledge (teachers' knowledge of content and students' thinking) to identify and diagnose students' misconceptions in addition and division of fractions. Teachers were expected to identify students' misconceptions, give reasons for these misconceptions and ask specific questions to diagnose students' thinking processes that lead to the misconceptions. A total of 40 teachers teaching mathematics were purposively selected from schools which performed poorly in the 2010 Basic Education Certificate Examinations in the Akatsi District. Questionnaires with four in-class problems consisting of students' commonest misconceptions in addition and division of fraction were used to collect data. Descriptive statistics was used to analyse the data. It was found that the majority of the teachers could identify students' misconceptions but could not articulate the reasons for such misconceptions clearly. It was also found that teachers could not ask specific questions to diagnose students' misconceptions. It was recommended that teacher training institutions integrate pedagogical content knowledge (teachers' knowledge of content and students' thinking) into the curriculum to equip teachers with skills that would enable them analyse students' thinking processes.

Key Words: pedagogical content knowledge, mathematics, fractions, errors — teachers'

Introduction

Teaching and learning of mathematics has attracted the attention of not only mathematics education researchers in Ghana but also the Government of Ghana. According to Anamuah-Mensah, Mereku and Ampiah (2008), junior high school form 2 students performed poorly in the 2004 Trends in Mathematics and Science Study (TIMSS) and when TIMSS data was analysed in the context for learning mathematics in Ghanaian schools, it was observed that students have little opportunity to use concepts, solve routine problems and reason mathematically.

Asiedu-Addo and Yidana (2000) discussed basic school pupils' poor performance in mathematics vis-a vis teachers' competence in Ghana and found that pupils' poor performance was due to teachers' low content knowledge and pedagogical knowledge in mathematics. Meanwhile, Darling-Hammond (2006) observed that quality teachers play a crucial role in developing students' knowledge, understanding and competencies needed to reach their full potential and contribute to their democratic society. Hence, the importance of quality teaching cannot be overemphasised since students need greater knowledge and skills to survive and succeed.

Kennedy (2001) was of the view that in order to achieve quality classroom teaching, the solutions should start from teachers. According to Kennedy (2001):

> *It is true that we want and need a 'quality profession'. Yet it is equally true that we need individual teachers who make up the profession to be committed to quality teaching..... what the profession says as a whole should come to life in individual classrooms (p. 6).*

Since 1989, international development partners such as the United States Agency for International Development (USAID), the Department for International Development (DFID) and Japan International Cooperation Agency (JICA) have embarked upon professional development programmes to improve the level of teaching in an effort to improve the quality of teaching which would in turn raise student achievement. Despite a decade of reforms, there is still concern that students' achievement in mathematics and science has not improved sufficiently to reflect the huge investment in basic education development (Ansu-Kyeremeh et al, 2002).

Ghanaian education researchers have also carried out research studies to reveal some of the difficulties pupils face learning mathematics by looking at teachers' content knowledge and pedagogical knowledge (Davis & Ampiah, 2008; Asiedu-Addo & Yidana, 2000). However, no study in Ghana has

particularly looked at teachers' pedagogical content knowledge in order to ascertain their knowledge of students' misconception or thinking processes, teachers' knowledge of reasons for students' misconception and how teachers ask questions to diagnose students' thought processes.

Literature Review

Literature has revealed that there is increasing evidence that teachers' knowledge of students' thought processes and reasoning during teaching and learning influences students' concept formation in mathematics (Gearheart& Saxe, 2004). Shulman (1986) pointed this out nearly 30 years ago when he specified the kinds of content knowledge teachers need beyond subject matter knowledge in order to teach their students effectively. Shulman defined pedagogical content knowledge as the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interest and abilities of learners, and presented for instruction.

Chick and Baker (2005) explored the usefulness of a framework for investigating the pedagogical content knowledge of two primary mathematics teachers who completed a questionnaire on Mathematics teaching and were later interviewed about their responses. Five solutions containing students' misunderstanding were given to the teachers to identify the students' misunderstanding and describe approaches they would use to teach subtraction of whole numbers, long divisions, addition of unlike fractions, decimals and ratio. It was found that the two teachers identified the students' misunderstanding but could not give the reasons for the students' misunderstanding.

Similarly, Oyunaa (2008) conducted a study on how mathematics teachers transform subject matter knowledge into classroom teaching. The study investigated two groups of teachers who had common professional degrees. The researcher categorized subject matter knowledge into content knowledge, pedagogical content knowledge and curriculum knowledge and their transformation in the classroom, focusing on fractions. Oyunaa pointed out that when teachers transform pedagogical content knowledge into classroom teaching, they emphasize one of the three categories of knowledge in accordance to the emphasized knowledge where the categories of knowledge are modified in the process to suit classroom teaching.

Turnuklu and Yesildere (2007) conducted a research involving 45 primary mathematics teachers. Four solutions containing students' misunderstanding were given in order to reveal the teachers' approaches to

teaching mathematics in topics of fractions, integers and decimals. Each of the problems fundamentally focused on teachers' interpretations of students' misconceptions or misunderstandings of Mathematical knowledge which was analysed quantitatively and qualitatively. It was found that teachers identified students' misconceptions but have difficulty in determining the reasons for the students' misconceptions about fractions and decimal fractions.

Ball, Mark, and Phleps (2007) explored teachers' pedagogical content knowledge, using an approach that was characterized as working 'bottom up'. They reported that teachers need both content and pedagogical knowledge to be able to organize classroom instruction, and present key ideas and concepts. This is because teachers could not give reasons for students' misunderstanding after identifying students' misconceptions.

Stacey, Steinle, Irwin, and Bana (2001) investigated teachers' content and pedagogical content knowledge of decimal numeration. Teachers were asked to complete decimal comparison test, mark items they thought would be difficult for students, and explain why it was difficult. It was recommended that teachers should emphasize content knowledge that integrates different aspects of number knowledge, and pedagogical content knowledge that includes thorough understanding of students' common difficulties.

Theoretical Framework

The study was based on Shulman's (1986) transformative theory of pedagogical content knowledge — *knowledge of content and students' thinking (KCS)*. Shulman identified that discipline knowledge alone is insufficient for successful teaching. According to Shulman (1986) teachers need a special knowledge of students' thinking to understand typical student conceptions, and why these conceptions exist. For that matter, Shulman presented a strong case for pedagogical content knowledge (PCK) as a specific form of knowledge for teaching which allows teachers to understand and anticipate particular preconceptions or learning difficulties of their students. Shulman's knowledge of content and students' thinking prepares teachers to be able to identify students' misconceptions, identify reasons responsible for students' misconceptions and diagnose students' thinking with appropriate questions. Indeed, Ball, Mark and Phleps (2008) reported that teachers needed the pedagogical content knowledge to be able to organize classroom instructions, and present key ideas and concepts to students because Mathematical knowledge required for teaching is indeed multidimensional.

Also, World Bank (2007) thematic study on Developing Science, Mathematics and Information Communication Technology Education in Sub-Saharan Africa (SMICT) with Ghana among the ten countries selected, indicated that inadequate pedagogical content knowledge of teachers in mathematics put limitations on the successful implementation of new curricular with intended new content and teaching methodology. It was identified that teachers did not have sufficient confidence in their ability to deal with matters if students went wrong during lessons.

In GES/ JICA (2008) survey on lesson observation, it was found that pupils' errors and mistakes were not well dealt with. This was because teachers teaching mathematics in basic schools lacked error analysis skills. According to the survey team, in order to improve classroom practice, teachers need to be equipped with not only subject content knowledge and teaching strategy but also pedagogical content knowledge which provides teachers with perspective as to how to use teaching and learning materials, how to enhance the learning of pupils and how to analyse pupils' mistakes since utilizing pupils' mistakes can be one of the most effective strategies of teaching and learning.

However, no specific research has been conducted as a follow up to confirm the observations made in World Bank (2007) thematic study and GES/ JICA (2008) survey on lesson observation. The present study intended to investigate teachers' pedagogical content knowledge vis-à-vis identification of pupils' misconceptions, giving reasons for pupils' misconceptions and asking specific questions to diagnose pupils' misconceptions in the teaching and learning of addition and division of fractions.

The purpose of the study was therefore to explore how junior high school teachers teaching Mathematics use their pedagogical content knowledge to identify pupils' misconceptions, give reasons for pupils' misconceptions and ask specific questions to diagnose pupils' misconceptions in the teaching and learning of addition and division of fractions.

Research Questions

The study intended to answer the following three questions:

1. How well do junior high school mathematics teachers use their pedagogical content knowledge to identify students' misconceptions in addition and division of fractions?
2. To what extent can teachers give reasons for pupils' misconceptions in addition and division of fractions?

3. How well do junior high school teachers use their pedagogical content knowledge to ask questions in order to diagnose students' misconceptions in addition and division of fractions?

Research Design

The research made use of descriptive survey design. This type of research would assist to describe the characteristics that exist in a population, but not to determine the cause-and- effect relationship. The justification for the use of the design was that it would provide detailed description of the professional competence of mathematics teachers and how these translated into the teaching and learning of addition and division of fractions.

Population and Sampling

The population for the study was 220 teachers in 35 junior high schools in the Akatsi District of the Volta Region. There were 10 circuits in the district from which eight (8) schools were purposively selected based on the schools' performance in the 2010 Basic Education Certificate Examinations provided by the Akatsi District Education Office. There were forty teachers in the selected schools for the study which comprised 38 males and 2 females. All the teachers in the selected schools took part in the study because they all taught mathematics.

Instrument

The instrument used for the study was a questionnaire. Some items in the questionnaire on the misconceptions of addition and division of fractions were adapted from a similar instrument used in a study conducted by Chick and Baker (2005). The questions adapted from Chick and Baker comprised four in-class problems which consisted of two questions each on addition and division of fractions. The four in-class problems were designed to investigate the use of teachers' PCK to identify students' misconceptions, give reasons for students' misconceptions, and ask specific questions to diagnose students' thinking processes.

The questionnaire was structured with open ended items. The complete questionnaire was given to a team of supervisors in the field of mathematics education for expert judgment in order to ensure content validity and item relevance. The questionnaire-approach was suitable because it allowed teachers to make considered responses to the questionnaire without feeling pressured to answer on the spot.

Pilot Study

The questionnaire was pilot tested at Abor Weme Junior High School for item relevance. This school was not part of the population for the study. Twelve mathematics teachers took part in the study. The teachers' responses led to the restructuring and modification of some of the items in the questionnaire. During the pilot study, teachers' responses were read to them and discussed in order to find out if what they wrote were exactly what they intended to write. This was to enable us compare their responses with the responses of the teachers in the main study.

Data Collection Procedure

The researchers visited the junior high schools selected and administered the questionnaires to forty (40) teachers who were teaching mathematics in the eight schools purposively selected. Face to face discussion was held before the questionnaires were administered to the teachers. There were twelve (12) items on teachers' pedagogical content knowledge in the questionnaire which were sub-divided into three thematic areas namely, identification of students' misconception, giving reason(s) for students' misconception, and diagnosing students' misconception through specific questioning.

Data Analysis

Data were analysed using frequencies and means. The frequencies for each of the criteria were set according to the components of pedagogical content knowledge. The criteria for each problem are listed below:

1) Identifying students' misconception
2) Giving appropriate reason(s) for students' misconception
3) Asking specific questions to diagnose students' misconception.

In the analyses, three (3) points each were given for correct identification of misconceptions, correct reasons for misconception, and asking specific questions (what the student did wrong) to diagnose students' misconception. Two (2) points each were given for providing insufficient reasons for students' misconceptions and asking leading questions (questions directing students towards correct answers) instead of specific questions to diagnose students' misconceptions and one (1) point each was given for only -identification of students' misconception, and asking unclear questions (ambiguous questions) to diagnose students' misconceptions.

Also, a grand mean between 2.45 – 3.00 was determined as excellent; a grand mean between 1.95 —2.44 as moderate and a grand mean between 1.00

scores were calculated and used for assessing their performance on all 4 problems combined and they were interpreted according to the three (3) criteria listed above.

Results

Research Question 1: How well do junior high school mathematics teachers use their pedagogical content knowledge to identify students' misconceptions in addition and division of fractions?

Table 1: Number and Percentage of Teachers' Responses Regarding Identification of Students' Misconceptions

Nature of Error	1 point	3 point	Mean score $(f_1x_1 + f_3x_3)/40$
Solution $\frac{3}{7} + \frac{2}{7} = \frac{5}{14}$	10 (25%)	30 (75%)	2.50
Solution $\frac{3}{8} + \frac{1}{9} = \frac{4}{17}$	15 (37.5%)	25(62.5%)	2.25
Solution $\frac{9}{10} \div \frac{3}{10} = \frac{10}{9} \div \frac{3}{10} = \frac{30}{90}$	16(40%)	24(60%	2.20
Solution $4 \div \frac{1}{4} = \frac{1}{4} \times 4 = 1$	18(45%)	22(55%)	2.10
Grand Mean	36.9%	63.1%	2.26

Where x_1 is the score 1, x_3 is the score 3 and f_i is the corresponding frequency.

Table 1 indicates that a large number of teachers (Mean = 63.1%) was able to identify students' misconceptions. This showed that most of the teachers were aware of students' misconceptions when carrying out operation on fractions. A grand mean of 2.26 indicated that on the average, the teachers used pedagogical content knowledge on identification of students' misconceptions reasonably well.

Research Question 2: To what extent can teachers give reasons for pupils' misconceptions in addition and division of fractions?

Table 2: Number and Percentage of Teachers' Responses Regarding Teachers' Knowledge of Reasons for Students' Misconceptions

Nature of Error	1 point for Inaccurate reason	2 points for Insufficient reason	3 points for Good reason	Mean score $(f_1x_1 + f_2x_{2+} f_3x_3)/40$
Solution $\frac{3}{7} + \frac{2}{7} = \frac{5}{14}$	18(45%)	1 (2.25%)	21(52.5%)	2.08
Solution $\frac{3}{8} + \frac{1}{9} =$	26 (65%)	2 (5%)	12 (30%)	1.65
Solution $\frac{9}{10} \div \frac{3}{10} = \frac{10}{9} \div \frac{3}{10} = \frac{30}{90}$	25 (62.5%)	1 (2.5%)	14(35%)	1.73
Solution 4 $\div \frac{1}{4} = \frac{1}{4} \times 4 = 1$	26 (65%)	2 (5%)	12 (30%)	1.65
Grand Mean	**59.4%**	**14.8%**	**36.9%**	**1.78**

Where x_1 is the score 1, x_2 is the score 2, x_3 is the score 3 and f_i is the corresponding frequency, Table 2 shows that 59.4% of the teachers could not give any accurate reasons for pupils' misconceptions, while 36.9% of the teachers were able to give sufficient reasons for pupils' misconceptions. However, 14.8% gave insufficient reasons for pupils' misconceptions. Even though most of the teachers (Mean = 63.1%) were able to identify students' misconceptions, it was evident from Table 2 that a greater proportion (74.2%: 59.4% + 14.8%) of the teachers were not able to give convincing reasons to support the reasons why students showed those incorrect thought processes. A grand mean of 1.78 revealed that on the average the teachers had insufficient pedagogical content knowledge on giving appropriate reason(s) for students' misconceptions.

Research Question 3: How well do junior high school teachers use their pedagogical content knowledge to ask questions in order to diagnose students' misconceptions in addition and division of fractions?

Table 3: Number and Percentage of Teachers' Responses Regarding Questions Teachers ask to Diagnose Students' Misconceptions

Nature of Error	1 point for asking unclear questions	2 points for asking leading questions	3 points for asking specific questions	Mean score $(f_1x_1 + f_2x_2 + f_3x_3)/40$
Solution $\frac{3}{7} + \frac{2}{7} = \frac{5}{14}$	15(37.5%)	20 (50%)	5(12.5%)	1.75
Solution $\frac{3}{8} + \frac{1}{9} = \frac{4}{17}$	16 (40%)	18 (45%)	6 (15%)	1.75
Solution $\frac{9}{10} \div \frac{3}{10} = \frac{10}{9} \div \frac{3}{10} = \frac{30}{90}$	10 (25%)	24 (60%)	6 (15%)	1.90
Solution $4 \div \frac{1}{4} = \frac{1}{4} \times 4 = 1$	6 (15%)	30 (75%)	4(10%)	1.95
Grand Mean	**29.4%**	**57.5%**	**13.1%**	**1.84**

Where x_1 is the score 1, x_2 is the score 2, x_3 is the score 3 and f_i is the corresponding frequency, Table 3 indicates that a large number of teachers (Mean =57.5%) asked leading questions which were giving clues to the correct answers. Indeed, leading questions could not enable teachers to ascertain what actually went wrong during the students' thought processes unlike specific questions which would reveal what actually went wrong during the students' thinking processes. Only a small number (Mean =13.1%) asked specific questions that could reveal the students' thinking process that led to the misconceptions. This was because from Table 2, a greater proportion (74.2%: 59.4% + 14.8%) of the teachers could not give sufficient reasons why students exhibited those incorrect thought processes. A grand mean of 1.84 in Table 3 shows that on the average the teachers have insufficient pedagogical content knowledge on asking specific questions to diagnose students' misconceptions.

Discussion

The results in Table 1 and Table 2 showed that teachers were able to identify students' misconceptions but could not articulate the reasons for the misconceptions. The finding is consistent with the study conducted by Stacey, Helme, Steinle, Baturo, Irwin, and Bana (2001) who recommended that

teacher education needed to emphasize content knowledge that integrates pedagogical content knowledge that includes thorough understanding of students' common difficulties. It is also in line with the study conducted by GES/JICA (2008) which revealed that teachers' inability to articulate clearly reasons for students' misconceptions could be attributed to lack of error analysis skills. Again, it is supported by Chick and Baker (2005) and Ball, Mark and Phleps (2008) who found in their studies that teachers did not have sufficient knowledge of the reasons for students' misconceptions. As a result, most of the teachers failed to give specific reasons behind students' wrong conception and/ or reasoning.

Furthermore, the finding is consistent with Turnuklu and Yesildere's (2007) study which also revealed that teachers have difficulty in determining reasons for students' misconceptions. The results in Table 3 also pointed out that teachers were asking leading questions instead of specific questions to diagnose students' misconceptions. This finding gave credence to the study conducted by Battey, Chan, Franke, and Webb (2009) who also found that substantial minority of teachers asked specific questions to elicit students' misconceptions.

Conclusions

Based on the findings and discussions that followed, it can be concluded that:

1. most teachers were able to identify students' misconception;
2. teachers could not give sufficient reasons for the students' misconceptions and
3. teachers could not ask specific questions to diagnose students' misconceptions.

Recommendations

Based on the discussion and conclusions, it was recommended that;

1. Teacher training institutions could integrate pedagogical content knowledge (teachers' knowledge of students' thinking) courses which would equip teacher trainees with skills to analyse students' thinking processes.
2. Through School-Based INSET (SBI) and Cluster-Based INSET (CBI), the Ministry of Education in collaboration with the Ghana Education Service should emphasise pedagogical content knowledge activities to equip teachers with skills to analyze students' thinking processes.
3. The Ministry of Education in collaboration with the Curriculum Research and Development Division (CRDD) should explicitly

incorporate the concepts of pedagogical content knowledge into the Mathematics curriculum to enable teachers gain better insights into pupils' thinking to enable them perform proper error analysis on pupils' misconceptions.

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